

An introduction to Post-Quantum Cryptography (PQC)

Jean-Christophe Deneuville

 $\langle i$ ean-christophe.deneuville@enac.fr $>$

Fall 2020

- 1 [What you've learnt so far \(should have\)](#page-2-0)
- 2 [Classical vs Quantum computing](#page-13-0)
- 3 [Two noticeable quantum algorithms \(and their impact over cryptography\)](#page-36-0)
- 4 [State-of-the-art quantum computers](#page-77-0)
- **5** [Possible alternatives](#page-117-0)
- 6 [Post-quantum cryptography](#page-128-0)

-
- [Two noticeable quantum algorithms \(and their impact over cryptography\)](#page-36-0)
- [State-of-the-art quantum computers](#page-77-0)
-
-

NH What is cryptography

- **NH** What is cryptography
- \blacksquare How it relates to information security

- **NHat is cryptography**
- \blacksquare How it relates to information security
- Examples of ancestral constructions (Scytale, Cæsar, Vigenere, ...)

- **What is cryptography**
- How it relates to information security
- Examples of ancestral constructions (Scytale, Cæsar, Vigenere, ...)
- **Their weaknesses/cryptanalysis (Al-Kindi (frequency analysis), Babbage, ...)**

- **NHAT** Is cryptography
- \blacksquare How it relates to information security
- Examples of ancestral constructions (Scytale, Cæsar, Vigenere, ...)
- **Their weaknesses/cryptanalysis (Al-Kindi (frequency analysis), Babbage, ...)**
- Recent symmetric constructions (OTP, Enigma, Sigaba, ...)

- **What is cryptography**
- \blacksquare How it relates to information security
- Examples of ancestral constructions (Scytale, Cæsar, Vigenere, ...)
- **Their weaknesses/cryptanalysis (Al-Kindi (frequency analysis), Babbage, ...)**
- Recent symmetric constructions (OTP, Enigma, Sigaba, ...)
- **Modern symmetric constructions (Stream/Block cipher, DES, AES, Blowfish, RC4, ...)**

- **What is cryptography**
- \blacksquare How it relates to information security
- Examples of ancestral constructions (Scytale, Cæsar, Vigenere, ...)
- **Their weaknesses/cryptanalysis (Al-Kindi (frequency analysis), Babbage, ...)**
- Recent symmetric constructions (OTP, Enigma, Sigaba, ...)
- **Modern symmetric constructions (Stream/Block cipher, DES, AES, Blowfish, RC4, ...)**
- Asymmetric cryptography (RSA, Diffie-Hellman, ElGamal, ...)

- **What is cryptography**
- \blacksquare How it relates to information security
- Examples of ancestral constructions (Scytale, Cæsar, Vigenere, ...)
- **Their weaknesses/cryptanalysis (Al-Kindi (frequency analysis), Babbage, ...)**
- Recent symmetric constructions (OTP, Enigma, Sigaba, ...)
- **Modern symmetric constructions (Stream/Block cipher, DES, AES, Blowfish, RC4, ...)**
- Asymmetric cryptography (RSA, Diffie-Hellman, ElGamal, ...)
- (Collision Resistant) Hash Functions (and birthday paradox)

- **What is cryptography**
- \blacksquare How it relates to information security
- Examples of ancestral constructions (Scytale, Cæsar, Vigenere, ...)
- **Their weaknesses/cryptanalysis (Al-Kindi (frequency analysis), Babbage, ...)**
- Recent symmetric constructions (OTP, Enigma, Sigaba, ...)
- **Modern symmetric constructions (Stream/Block cipher, DES, AES, Blowfish, RC4, ...)**
- Asymmetric cryptography (RSA, Diffie-Hellman, ElGamal, ...)
- (Collision Resistant) Hash Functions (and birthday paradox)
- Digital Signatures (RSA, DSA, ECDSA, ...)

- **What is cryptography**
- \blacksquare How it relates to information security
- Examples of ancestral constructions (Scytale, Cæsar, Vigenere, ...)
- **Their weaknesses/cryptanalysis (Al-Kindi (frequency analysis), Babbage, ...)**
- Recent symmetric constructions (OTP, Enigma, Sigaba, ...)
- **Modern symmetric constructions (Stream/Block cipher, DES, AES, Blowfish, RC4, ...)**
- Asymmetric cryptography (RSA, Diffie-Hellman, ElGamal, ...)
- (Collision Resistant) Hash Functions (and birthday paradox)
- Digital Signatures (RSA, DSA, ECDSA, ...)
- Security models

- 1 [What you've learnt so far \(should have\)](#page-2-0)
- 2 [Classical vs Quantum computing](#page-13-0)
- [Two noticeable quantum algorithms \(and their impact over cryptography\)](#page-36-0)
- [State-of-the-art quantum computers](#page-77-0)
-
-

Classical computing

A classical computer (Turing machine) processes (through a language) classical boolean circuits.

The quantity of information is measured through Shannon's entropy, data can eventually be compressed, and there exist efficient algorithms for error correction.

Some circuits are computable *i.e.* the machine eventually halts (*e.g.* primality problem), some others aren't $(e.g.$ the halting problem).

Current security

Current security vs. classical computing power (2020)

Current security vs. classical computing power (2020)

1 standard machine: 64 bits architecture 2 6

1 standard machine: 8 cores $2^6 \times 2^3$

1 standard machine: 4 GHz $2^6 \times 2^3 \times 2^2 \times 10^9$

Current security vs. classical computing power (2020)

1 standard machine: running 1 month $2^6 \times 2^3 \times 2^2 \times 10^9 \times 60 \times 60 \times 24 \times 30$

Current security vs. classical computing power (2020)

NSA > 10 000 standard machines? $2^6 \times 2^3 \times 2^2 \times 10^9 \times 60 \times 60 \times 24 \times 30 \times 10^4$

Current security vs. classical computing power (2020)

NSA > 10 000 standard machines?

 $2^6 \times 2^3 \times 2^2 \times 10^9 \times 60 \times 60 \times 24 \times 30 \times 10^4 \approx 2^{80}$ elementary operations

 $NSA > 10000$ standard machines? (without possible GPU, ASICS, ...) $2^6 \times 2^3 \times 2^2 \times 10^9 \times 60 \times 60 \times 24 \times 30 \times 10^4 \approx 2^{80}$ elementary operations

 $NSA > 10000$ standard machines? (without possible GPU, ASICS, ...) $2^6 \times 2^3 \times 2^2 \times 10^9 \times 60 \times 60 \times 24 \times 30 \times 10^4 \approx 2^{80}$ elementary operations

A concrete example

During 2018, there were 2^{89} SHA-256 hashes computed on the blockchain BitCoin...

 $NSA > 10000$ standard machines? (without possible GPU, ASICS, ...) $2^6 \times 2^3 \times 2^2 \times 10^9 \times 60 \times 60 \times 24 \times 30 \times 10^4 \approx 2^{80}$ elementary operations

A concrete example

During 2018, there were 2^{89} SHA-256 hashes computed on the blockchain BitCoin...

Security in 2020

Setting parameters so that best known attacks have complexity (at least) $2^{128}.$

 $NSA > 10000$ standard machines? (without possible GPU, ASICS, ...) $2^6 \times 2^3 \times 2^2 \times 10^9 \times 60 \times 60 \times 24 \times 30 \times 10^4 \approx 2^{80}$ elementary operations

A concrete example

During 2018, there were 2^{89} SHA-256 hashes computed on the blockchain BitCoin...

Security in 2020

Setting parameters so that best known attacks have complexity (at least) $2^{128}.$

Classical best known attacks:

- Symmetric primitives: brute-force
- **Asymmetric primitives: GNFS, sub-exponential complexity**

Quantum computing has been suggested by Paul Benioff in 1980. He proposed a quantum mechanical model of the Turing machine, using two ingredients:

Quantum computing has been suggested by Paul Benioff in 1980. He proposed a quantum mechanical model of the Turing machine, using two ingredients:

Superposition: while a bit can be either in a state 0 or 1, a quantum bit (*qubit*) can be in any superposition of states $|0\rangle$ and $|1\rangle$.

Quantum computing has been suggested by Paul Benioff in 1980. He proposed a quantum mechanical model of the Turing machine, using two ingredients:

- Superposition: while a bit can be either in a state 0 or 1, a quantum bit (qubit) can be in any superposition of states $|0\rangle$ and $|1\rangle$.
- **Entanglement: the capability of two qubits to be correlated. If Alice and Bob both get one** of two entangled qubits, and if Alice measures a $|0\rangle$ at some point, then necessarily Bob must measure the same, as $|00\rangle$ is the only state where Alice's qubit is a $|0\rangle$.

Quantum computing has been suggested by Paul Benioff in 1980. He proposed a quantum mechanical model of the Turing machine, using two ingredients:

Superposition: while a bit can be either in a state 0 or 1, a quantum bit (qubit) can be in any superposition of states $|0\rangle$ and $|1\rangle$.

Entanglement: the capability of two qubits to be correlated. If Alice and Bob both get one of two entangled qubits, and if Alice measures a $|0\rangle$ at some point, then necessarily Bob must measure the same, as $|00\rangle$ is the only state where Alice's qubit is a $|0\rangle$.

Qubits can be "implemented" using the spin of an electron, or the polarization of a photon, ...

Quantum computing

As a consequence:

a vector of n entangled qubits can be in a superposition of any 2^n possible states at the same time,

As a consequence:

- a vector of n entangled qubits can be in a superposition of any 2^n possible states at the same time,
- against 1 among the 2^n possible states for a classical vector of n bits.

As a consequence:

- a vector of n entangled qubits can be in a superposition of any 2^n possible states at the same time,
- against 1 among the 2^n possible states for a classical vector of n bits.
- It is however not possible to observe these states all together at the same time.

As a consequence:

- a vector of n entangled qubits can be in a superposition of any 2^n possible states at the same time,
- against 1 among the 2^n possible states for a classical vector of n bits.

It is however not possible to observe these states all together at the same time. A quantum algorithm solving a problem needs to make the correct solution (state) exponentially more likely than the other states (cf. quantum annealing / wave function collapsing).

Outline

- 1 [What you've learnt so far \(should have\)](#page-2-0)
- 2 [Classical vs Quantum computing](#page-13-0)
- 3 [Two noticeable quantum algorithms \(and their impact over cryptography\)](#page-36-0)
- [State-of-the-art quantum computers](#page-77-0)
-
-

Shor's algorithm

SIAM J. COMPHT Vol. 26, No. 5, pp. 1484–1509. October 1997 C 1997 Society for Industrial and Applied Mathematics 000

POLYNOMIAL-TIME ALGORITHMS FOR PRIME FACTORIZATION AND DISCRETE LOGARITHMS ON A QUANTUM COMPUTER*

PETER W SHORT

Abstract. A digital computer is generally believed to be an efficient universal computing device: that is, it is believed able to simulate any physical computing device with an increase in computation time by at most a polynomial factor. This may not be true when quantum mechanics is taken into consideration. This paper considers factoring integers and finding discrete logarithms, two problems which are generally thought to be hard on a classical computer and which have been used as the basis of several proposed cryptosystems. Efficient randomized algorithms are given for these two problems on a hypothetical quantum computer. These algorithms take a number of steps polynomial in the input size, e.g., the number of digits of the integer to be factored.

Algorithm 1: ShorAlgorithm (N)

Input: N **Output:** p, q such that $N = pq$

Algorithm 1: ShorAlgorithm (N)

Input: N **Output:** p, q such that $N = pq$ 1 Pick $g \in \mathbb{Z}_N$ at random;

Algorithm 1: ShorAlgorithm (N)

Input: N **Output:** p, q such that $N = pq$ 1 Pick $q \in \mathbb{Z}_N$ at random; 2 if $gcd(q, N) \neq 1$ then 3 then return $(p = \gcd(g, N), q = N/p)$

Algorithm 1: ShorAlgorithm (N)

Input: N **Output:** p, q such that $N = pq$ 1 Pick $q \in \mathbb{Z}_N$ at random; 2 if $gcd(q, N) \neq 1$ then 3 then return $(p = \gcd(g, N), q = N/p)$ 4 Find r such that $g^r \equiv 1 [N];$

Algorithm 1: ShorAlgorithm (N)

Input: N **Output:** p, q such that $N = pq$ 1 Pick $g \in \mathbb{Z}_N$ at random; 2 if $gcd(q, N) \neq 1$ then 3 then return $(p = \gcd(g, N), q = N/p)$ 4 Find r such that $g^r \equiv 1 [N];$ 5 if $r \equiv 0[2]$ then 6 **return** gcd $(g^{r/2} \pm 1, N)$ ⁷ else ⁸ go to [1](#page-38-1)

Shor's algorithm: how it works

"Find r such that $g^r \equiv 1 [N];$ "

[Post-quantum cryptography](#page-0-0) / [Remarkable quantum algorithms](#page-43-0) 14

Shor's algorithm: how it works

"Find r such that $g^r \equiv 1 [N];$ "

First question: How does finding r such that $g^r \equiv 1 [N]$ help factoring?

Shor's algorithm: how it works

"Find r such that $g^r \equiv 1 [N];$ "

First question: How does finding r such that $g^r \equiv 1 [N]$ help factoring?

 $g^r \equiv 1[N] \iff \exists k \in \mathbb{N}^*$ such that $g^r = kN + 1$

Shor's algorithm: how it works

"Find r such that $g^r \equiv 1 [N];$ "

First question: How does finding r such that $g^r \equiv 1 [N]$ help factoring?

$$
g^r \equiv 1[N] \Leftrightarrow \exists k \in \mathbb{N}^* \text{ such that } g^r = kN + 1
$$

$$
\Leftrightarrow g^r - 1 = kN
$$

"Find r such that $g^r \equiv 1 [N];$ "

First question: How does finding r such that $g^r \equiv 1 [N]$ help factoring?

$$
g^r \equiv 1[N] \quad \Leftrightarrow \quad \exists k \in \mathbb{N}^* \text{ such that } g^r = kN + 1
$$

$$
\Leftrightarrow \quad g^r - 1 = kN
$$

$$
\text{(assuming r is even)} \quad \Leftrightarrow \quad \left(g^{r/2} - 1\right)\left(g^{r/2} + 1\right) = kN
$$

"Find r such that $g^r \equiv 1 [N];$ "

First question: How does finding r such that $g^r \equiv 1 [N]$ help factoring?

$$
g^r \equiv 1[N] \quad \Leftrightarrow \quad \exists k \in \mathbb{N}^* \text{ such that } g^r = kN + 1
$$

$$
\Leftrightarrow \quad g^r - 1 = kN
$$

$$
\text{(assuming r is even)} \quad \Leftrightarrow \quad \left(g^{r/2} - 1\right)\left(g^{r/2} + 1\right) = kN
$$

Meaning that there is a non-negligible probability that $g^{r/2} \pm 1$ shares non trivial factors with N .

Shor's algorithm: how it works

Example with $N = 314191$, find p, q (source: [minutephysics\)](https://www.youtube.com/watch?v=FRZQ-efABeQ)

Example with $N = 314191$, find p, q (source: [minutephysics\)](https://www.youtube.com/watch?v=FRZQ-efABeQ) (source: minutephysics)

■ step 1. $g \leftarrow 101$

Example with $N = 314191$, find p, q (source: [minutephysics\)](https://www.youtube.com/watch?v=FRZQ-efABeQ) (source: minutephysics)

- step 1. $g \leftarrow 101$
- step 2. $r \leftarrow 4347$

Example with $N = 314191$, find p, q

- step 1. $g \leftarrow 101$
- step 2. $r \leftarrow 4347$
- step 3. r is odd... go to [1](#page-38-1)

Example with $N = 314191$, find p, q

- step 1. $q \leftarrow 101$
- step 2. $r \leftarrow 4347$
- **step 3.** r is **odd...** go to [1](#page-38-1)
- step 1. $q \leftarrow 127$

Example with $N = 314191$, find p, q

- step 1. $q \leftarrow 101$
- step 2. $r \leftarrow 4347$
- **step 3.** r is **odd...** go to [1](#page-38-1)
- step 1. $q \leftarrow 127$
- step 2. $r \leftarrow 17388$

Example with $N = 314191$, find p, q

- step 1. $q \leftarrow 101$
- step 2. $r \leftarrow 4347$
- **step** 3. r is **odd**... go to [1](#page-38-1)
- step 1. $q \leftarrow 127$
- step 2. $r \leftarrow 17388$
- step 3. let us denote $g_p=g^{17388/2}+1$ and $g_q=g^{17388/2}-1$

Example with $N = 314191$, find p, q

- step 1. $q \leftarrow 101$
- step 2. $r \leftarrow 4347$
- **step** 3. r is **odd**... go to [1](#page-38-1)
- step 1. $q \leftarrow 127$
- step 2. $r \leftarrow 17388$
- step 3. let us denote $g_p=g^{17388/2}+1$ and $g_q=g^{17388/2}-1$ we have that $gcd(q_n, N) = 829 =: p$ and $gcd(q_a, N) = 379 =: q$ and indeed, $p \cdot q = 829 \times 379 = 314191 = N$

Second question: "Wait a minute, I was expecting some magic quantum trick out there. Where the is the quantum part?"

Second question: "Wait a minute, I was expecting some magic quantum trick out there. Where the is the quantum part?"

"Find r such that $g^r \equiv 1 [N];$ "

Second question: "Wait a minute, I was expecting some magic quantum trick out there. Where the is the quantum part?"

"Quantumly find r such that $g^r \equiv 1 [N];$ "

Second question: "Wait a minute, I was expecting some magic quantum trick out there. Where the is the quantum part?"

"Quantumly find r such that $g^r \equiv 1 [N];$ "

The complexity to find the *period* of the function $g \mapsto g^x \mod N$ is:

Second question: "Wait a minute, I was expecting some magic quantum trick out there. Where the is the quantum part?"

"Quantumly find r such that $g^r \equiv 1 [N];$ "

The complexity to find the *period* of the function $g \mapsto g^x \mod N$ is:

Classically $\mathcal{O}(N)$

Second question: "Wait a minute, I was expecting some magic quantum trick out there. Where the is the quantum part?"

"Quantumly find r such that $g^r \equiv 1 [N];$ "

The complexity to find the *period* of the function $g \mapsto g^x \mod N$ is:

- Classically $\mathcal{O}(N)$
- Quantumly $\mathcal{O}\left(\log\left(N\right)^3\right)$

Second question: "Wait a minute, I was expecting some magic quantum trick out there. Where the is the quantum part?"

"Quantumly find r such that $g^r \equiv 1 [N];$ "

The complexity to find the *period* of the function $g \mapsto g^x \mod N$ is:

- **Classically** $\mathcal{O}(N)$
- Quantumly $\mathcal{O}\left(\log\left(N\right)^3\right)$. That's an exponential speedup!

[Post-quantum cryptography](#page-0-0) / [Remarkable quantum algorithms](#page-63-0) 16

Quantum period finding

How does it work? Why is it much much faster quantumly?

Fourier Transform is THE tool to analyse frequencies. Fortunately, it has a quantum equivalent: QFT.

Quantum computing allows to provide QFT a superposition of every possible states (assuming enough qubits).

Factoring becomes polynomial-time $\mathcal{O}\left(\left(\log N\right)^2\left(\log\log N\right)\left(\log\log\log N\right)\right)$

Factoring becomes polynomial-time $\mathcal{O}\left(\left(\log N\right)^2\left(\log\log N\right)\left(\log\log\log N\right)\right)$ against $\exp (1.9(\log N)^{1/3}(\log \log N)^{2/3})$ classically

Factoring becomes polynomial-time $\mathcal{O}\left(\left(\log N\right)^2\left(\log\log N\right)\left(\log\log\log N\right)\right)$ against $\exp (1.9(\log N)^{1/3}(\log \log N)^{2/3})$ classically

- **Factoring becomes polynomial-time** $\mathcal{O}\left(\left(\log N\right)^2\left(\log\log N\right)\left(\log\log\log N\right)\right)$ against $\exp (1.9(\log N)^{1/3}(\log \log N)^{2/3})$ classically
- Discrete logarithm becomes almost polynomial-time

- **Factoring becomes polynomial-time** $\mathcal{O}\left(\left(\log N\right)^2\left(\log\log N\right)\left(\log\log\log N\right)\right)$ against $\exp (1.9(\log N)^{1/3}(\log \log N)^{2/3})$ classically
- Discrete logarithm becomes almost polynomial-time

No more RSA, DSA, ECDSA, ElGamal, ...

- **Factoring becomes polynomial-time** $\mathcal{O}\left(\left(\log N\right)^2\left(\log\log N\right)\left(\log\log\log N\right)\right)$ against $\exp (1.9(\log N)^{1/3}(\log \log N)^{2/3})$ classically
- Discrete logarithm becomes almost polynomial-time

```
No more RSA, DSA, ECDSA, ElGamal, ...
```


In other words, security as we know it collapses...

Grover's algorithm

A fast quantum mechanical algorithm for database search

Lov K. Grover 3C-404A. Bell Labs 600 Mountain Avenue Murray Hill NJ 07974 lkgrover@bell-labs.com

Summary

Imagine a phone directory containing N names arranged in completely random order. In order to find someone's phone number with a probability of $\frac{1}{2}$, any classical algorithm (whether deterministic or probabilistic) will need to look at a minimum of $\frac{N}{2}$ names. Quantum mechanical systems can be in a superposition of states and simultaneously examine multiple names. By properly adjusting the phases of various operations, successful computations reinforce each other while others interfere randomly. As a result, the desired phone number can be obtained in only $O(\sqrt{N})$ steps. The algorithm is within a small constant factor of the fastest possible quantum mechanical algorithm.

(*n*-entries unsorted) Database search takes $\mathcal{O}\left(\sqrt{n}\right)$ queries instead of $\mathcal{O}\left(n\right)$.

(*n*-entries unsorted) Database search takes $\mathcal{O}\left(\sqrt{n}\right)$ queries instead of $\mathcal{O}\left(n\right)$.

Consequence over symmetric crypto:

(*n*-entries unsorted) Database search takes $\mathcal{O}\left(\sqrt{n}\right)$ queries instead of $\mathcal{O}\left(n\right)$.

Consequence over symmetric crypto:

 \rightarrow The length of the secret key must be **doubled** to preserve the same level of security

(*n*-entries unsorted) Database search takes $\mathcal{O}\left(\sqrt{n}\right)$ queries instead of $\mathcal{O}\left(n\right)$.

Consequence over symmetric crypto:

 \rightarrow The length of the secret key must be **doubled** to preserve the same level of security

Consequence over hash functions:

(*n*-entries unsorted) Database search takes $\mathcal{O}\left(\sqrt{n}\right)$ queries instead of $\mathcal{O}\left(n\right)$.

Consequence over symmetric crypto:

 \rightarrow The length of the secret key must be **doubled** to preserve the same level of security

Consequence over hash functions:

 \rightarrow More tricky (depending on the model, the size of the quantum computer, ...), at least $+33\%$ to preserve the security level

Outline

- 1 [What you've learnt so far \(should have\)](#page-2-0)
-
- [Two noticeable quantum algorithms \(and their impact over cryptography\)](#page-36-0)
- 4 [State-of-the-art quantum computers](#page-77-0)
-
-

How far are we from a large-scale quantum computer?

10.000

A quantum analog to Moore's law: the number of qubits (y -axe) approximately doubles every year (x -axe). (Source: D-Wave)

This analog to Moore's law has several drawbacks:

This analog to Moore's law has several drawbacks:

Example 1 essentially corresponds to multiple 32 qubits architectures mounted in parallel

This analog to Moore's law has several drawbacks:

- **Example 1** essentially corresponds to multiple 32 qubits architectures mounted in parallel
- **fault-tolerance remains an open problem**

This analog to Moore's law has several drawbacks:

- **Example 1** essentially corresponds to multiple 32 qubits architectures mounted in parallel
- **fault-tolerance remains an open problem**
- still far from what is required to factor 2048 bits moduli

This analog to Moore's law has several drawbacks:

- **EXECUTE:** essentially corresponds to multiple 32 qubits architectures mounted in parallel
- **fault-tolerance remains an open problem**
- still far from what is required to factor 2048 bits moduli

In 2020, the largest quantum computer features 72 qubits (Google).

 ${\sf A}$ bit of perspective regarding quantum stuffs. $(\sf {full~story~here})$

 ${\sf A}$ bit of perspective regarding quantum stuffs. $(\sf {full~story~here})$

1968: Wiesner describes conjugate (quantum) coding

A bit of perspective regarding quantum stuffs. (full story [here\)](https://en.wikipedia.org/wiki/Timeline_of_quantum_computing_and_communication)

- 1968: Wiesner describes conjugate (quantum) coding
- 1984: Bennett & Brassard use Wiesner's coding for QKD

A bit of perspective regarding quantum stuffs. (full story [here\)](https://en.wikipedia.org/wiki/Timeline_of_quantum_computing_and_communication)

- 1968: Wiesner describes conjugate (quantum) coding
- 1984: Bennett & Brassard use Wiesner's coding for QKD
- 1994: Shor's algorithm

- A bit of perspective regarding quantum stuffs. (full story [here\)](https://en.wikipedia.org/wiki/Timeline_of_quantum_computing_and_communication)
	-

- 1968: Wiesner describes conjugate (quantum) coding
- 1984: Bennett & Brassard use Wiesner's coding for QKD
- 1994: Shor's algorithm
- 1995: 1st US DoD workshop on Quantum Crypto / Shor proposes quantum error correction

- A bit of perspective regarding quantum stuffs. (full story [here\)](https://en.wikipedia.org/wiki/Timeline_of_quantum_computing_and_communication)
	-

- 1968: Wiesner describes conjugate (quantum) coding
- 1984: Bennett & Brassard use Wiesner's coding for QKD
- 1994: Shor's algorithm
- 1995: 1st US DoD workshop on Quantum Crypto / Shor proposes quantum error correction
- 1996: Grover's algorithm

- A bit of perspective regarding quantum stuffs. (full story [here\)](https://en.wikipedia.org/wiki/Timeline_of_quantum_computing_and_communication)
	-

- 1968: Wiesner describes conjugate (quantum) coding
- 1984: Bennett & Brassard use Wiesner's coding for QKD
- 1994: Shor's algorithm
- 1995: 1st US DoD workshop on Quantum Crypto / Shor proposes quantum error correction
- 1996: Grover's algorithm
- 2001: First execution of Shor's algorithm, factoring 15

- A bit of perspective regarding quantum stuffs. (full story [here\)](https://en.wikipedia.org/wiki/Timeline_of_quantum_computing_and_communication)
	-

- 1968: Wiesner describes conjugate (quantum) coding
- 1984: Bennett & Brassard use Wiesner's coding for QKD
- 1994: Shor's algorithm
- 1995: 1st US DoD workshop on Quantum Crypto / Shor proposes quantum error correction
- 1996: Grover's algorithm
- 2001: First execution of Shor's algorithm, factoring 15
- 2002: Creation of the Institute for Quantum Computing

- A bit of perspective regarding quantum stuffs. (full story [here\)](https://en.wikipedia.org/wiki/Timeline_of_quantum_computing_and_communication)
- 1968: Wiesner describes conjugate (quantum) coding
- 1984: Bennett & Brassard use Wiesner's coding for QKD
- 1994: Shor's algorithm
- 1995: 1st US DoD workshop on Quantum Crypto / Shor proposes quantum error correction
- 1996: Grover's algorithm
- 2001: First execution of Shor's algorithm, factoring 15
- 2002: Creation of the Institute for Quantum Computing
- 2003: DARPA's quantum network operational

- A bit of perspective regarding quantum stuffs. $(f_{\text{full story here}})$
- 1968: Wiesner describes conjugate (quantum) coding
- 1984: Bennett & Brassard use Wiesner's coding for QKD
- 1994: Shor's algorithm
- 1995: 1st US DoD workshop on Quantum Crypto / Shor proposes quantum error correction
- 1996: Grover's algorithm
- 2001: First execution of Shor's algorithm, factoring 15
- 2002: Creation of the Institute for Quantum Computing
- 2003: DARPA's quantum network operational
- 2009: qubits lifetime $\simeq 100$ ms

- A bit of perspective regarding quantum stuffs. $(f_{\text{full story here}})$
- 1968: Wiesner describes conjugate (quantum) coding
- 1984: Bennett & Brassard use Wiesner's coding for QKD
- 1994: Shor's algorithm
- 1995: 1st US DoD workshop on Quantum Crypto / Shor proposes quantum error correction
- 1996: Grover's algorithm
- 2001: First execution of Shor's algorithm, factoring 15
- 2002: Creation of the Institute for Quantum Computing
- 2003: DARPA's quantum network operational
- 2009: qubits lifetime $\simeq 100$ ms
- 2012: D-Wave claims a quantum computation using 84 qubits (24 computational)

- A bit of perspective regarding quantum stuffs. $(f_{\text{full story here}})$
- 1968: Wiesner describes conjugate (quantum) coding
- 1984: Bennett & Brassard use Wiesner's coding for QKD
- 1994: Shor's algorithm
- 1995: 1st US DoD workshop on Quantum Crypto / Shor proposes quantum error correction
- 1996: Grover's algorithm
- 2001: First execution of Shor's algorithm, factoring 15
- 2002: Creation of the Institute for Quantum Computing
- 2003: DARPA's quantum network operational
- 2009: qubits lifetime $\simeq 100$ ms
- 2012: D-Wave claims a quantum computation using 84 qubits (24 computational)
- 2013: coherence time 39mins at room temperature / 1^{st} Snowden revelations

- A bit of perspective regarding quantum stuffs. $(f_{\text{full story here}})$
- 1968: Wiesner describes conjugate (quantum) coding
- 1984: Bennett & Brassard use Wiesner's coding for QKD
- 1994: Shor's algorithm
- 1995: 1st US DoD workshop on Quantum Crypto / Shor proposes quantum error correction
- 1996: Grover's algorithm
- 2001: First execution of Shor's algorithm, factoring 15
- 2002: Creation of the Institute for Quantum Computing
- 2003: DARPA's quantum network operational
- 2009: qubits lifetime $\simeq 100$ ms
- 2012: D-Wave claims a quantum computation using 84 qubits (24 computational)
- 2013: coherence time 39mins at room temperature / 1^{st} Snowden revelations
- 2014: NSA project "Penetrating Hard Target" aims at breaking quantumly strong crypto

- A bit of perspective regarding quantum stuffs. $(f_{\text{full story here}})$
- 1968: Wiesner describes conjugate (quantum) coding
- 1984: Bennett & Brassard use Wiesner's coding for QKD
- 1994: Shor's algorithm
- 1995: 1st US DoD workshop on Quantum Crypto / Shor proposes quantum error correction
- 1996: Grover's algorithm
- 2001: First execution of Shor's algorithm, factoring 15
- 2002: Creation of the Institute for Quantum Computing
- 2003: DARPA's quantum network operational
- 2009: qubits lifetime $\simeq 100$ ms
- 2012: D-Wave claims a quantum computation using 84 qubits (24 computational)
- 2013: coherence time 39mins at room temperature / 1^{st} Snowden revelations
- 2014: NSA project "Penetrating Hard Target" aims at breaking quantumly strong crypto
- 2015: NSA statement / 2016: NIST preparing PQC standardization

- A bit of perspective regarding quantum stuffs. $(f_{\text{full story here}})$
- 1968: Wiesner describes conjugate (quantum) coding
- 1984: Bennett & Brassard use Wiesner's coding for QKD
- 1994: Shor's algorithm
- 1995: 1st US DoD workshop on Quantum Crypto / Shor proposes quantum error correction
- 1996: Grover's algorithm
- 2001: First execution of Shor's algorithm, factoring 15
- 2002: Creation of the Institute for Quantum Computing
- 2003: DARPA's quantum network operational
- 2009: qubits lifetime $\simeq 100$ ms
- 2012: D-Wave claims a quantum computation using 84 qubits (24 computational)
- 2013: coherence time 39mins at room temperature / 1^{st} Snowden revelations
- 2014: NSA project "Penetrating Hard Target" aims at breaking quantumly strong crypto
- 2015: NSA statement / 2016: NIST preparing PQC standardization
- 2017: industry race for largest quantum computer / D-Wave 2000Q / NIST PQC starts

- A bit of perspective regarding quantum stuffs. $(f_{\text{full story here}})$
- 1968: Wiesner describes conjugate (quantum) coding
- 1984: Bennett & Brassard use Wiesner's coding for QKD
- 1994: Shor's algorithm
- 1995: 1st US DoD workshop on Quantum Crypto / Shor proposes quantum error correction
- 1996: Grover's algorithm
- 2001: First execution of Shor's algorithm, factoring 15
- 2002: Creation of the Institute for Quantum Computing
- 2003: DARPA's quantum network operational
- 2009: qubits lifetime $\simeq 100$ ms
- 2012: D-Wave claims a quantum computation using 84 qubits (24 computational)
- 2013: coherence time 39mins at room temperature / 1^{st} Snowden revelations
- 2014: NSA project "Penetrating Hard Target" aims at breaking quantumly strong crypto
- 2015: NSA statement / 2016: NIST preparing PQC standardization
- 2017: industry race for largest quantum computer / D-Wave 2000Q / NIST PQC starts
- 2018: Google announces a 72-qubit quantum chip / 2019: quantum supremacy

Ecole Nationale de l'Aviation Civile

Hot news!

Ecole Nationale de l'Aviation Civile

Google's supposed milestone achievement became public last month when a
preprint scientific paper accidentally leaked on the website. exerting the supposed milestone achievement became public last month when a
preprint scientific paper accidentally leaked on the website of NASA, a collaborator,
as *Fortune* reported at the time. Google has said nothing a ²
as *Fortune* reported at the time. Google has said nothievebsite of NASA, a collabor
historic experiment since then, lending credence to which and the potentially
bound to silenge. historic experiment since then, lending credence to whispers that its researchers
bound to silence under the terms of a news embargo has said nothing shout the potentially
bound to silence under the terms of a news embargo bound to silence under the terms of a news embargo by a maple to disclose more information until a certain data with the searchers
unable to disclose more information until a certain data with the science journal,
imminent anable to suchce under the terms of a news embargo by a major science journal,
unable to disclose more information until a certain date which is presumed to be
imminent.

[Post-quantum cryptography](#page-0-0) / [State-of-the-art quantum computers](#page-103-0) 25

Ecole Nationale de l'Aviation Civile

What is quantum supremacy?

What is quantum supremacy?

Quantum supremacy refers to the moment where a functional quantum computer can effectively solve a problem that is not solvable (within decent time frame, $e.g.$ 100 years) with any (super) computer.

129

What why is Google's quantum supremacy experiment impressive?

In the Nature naner nublished by Google, they say **A**

To demonstrate quantum supremacy, we compare our quantum processor against state-of-the-art classical Quantum supremacy in the task of sampling the couput of a pseudo-random quantum circuit. Random circuits are a

Computer can function computation and the computational hardness. We design the circuits to entangle a set of $\left\{\begin{array}{rcl} \text{effectively solve} & \text{squareness of computation and hardness in the circuit, is a small amplitude of quantum circuit, is not possible to be a small amplitude of the circuit.} \\ \text{of the total amplitude of significant deviation of the amount of time.} & \text{of the amount of interest, the amount of time.} \\ \text{of the total value of the amount of time.} & \text{of the time of the circuit.} \end{array} \right. \\ \left\{\begin{array}{rcl} \text{from $e.g.$ 100 years} & \text{with} \\ \text{of the total value of the amount of time.} \\ \text{of the total value of the time.} \end{array} \right. \\ \left\{\begin{array}{rcl} \text{of the total value of the time.} \\ \text{$ any (super) contract a sec or pushings, for example (1000101), 1011100, Owing to quantum interteferice, the
any order of the bistsripation of the bistsripation of the bistrings resembles a speckled intersting partic this probability distribution becomes exponentially more difficult as the number of qubits (width) and number of gate cycles (denth) grow.

> So, from what I can tell, they configure their qubits into a pseudo-randomly generated circuit, which, when run. buts the qubits into a state vector that represents a probability distribution over 2⁵³ possible states of the qubits. but that distribution is intractable to calculate, or even estimate via sampling using a classical computer simulation. But they sample it by "looking" at the state of the qubits after running the circuit many times.

Isn't this just an example of creating a system whose output is intractable to calculate, and then "calculating" it by simply observing the output of the system?

It sounds similar to saying:

If I spill this pudding cup on the floor, the exact pattern it will form is very chaotic, and intractable for any supercomputer to calculate. But I just invented a new special type of computer: this pudding cup. And I'm going to do the calculation by spilling it on the floor and observing the result. I have achieved pudding supremacy.

What why is Google's quantum supremacy experiment impressive?

Asked 13 days ago Active 11 days ago Mewed 12k times

In the Nature naner published by Google, they say Δ

129

To demonstrate quantum supremacy, we compare our quantum processor against state-of-the-art classical $\begin{array}{|l|c|c|}\hline \textbf{Quantum} & \textbf{sup} & \textbf{non-pu} \\ \hline \textbf{quantum} & \textbf{sup} & \textbf{non-pu} \\ \hline \textbf{quantum} & \textbf{non-pu} & \textbf{non-pu} \\ \hline \textbf{normal} & \textbf{non-pu} & \textbf{non-pu} \\ \hline \textbf{normal} &$ effectively solve $z \rightarrow z$ problem that is not computational hardness. We design the circuits to entangle a set of quantum bits (qubits) by \blacksquare
encoding a problem that is not solve that is not solve that is not contained any (super) computer and account of the bistricity of the contract processes apprecise is appected intensity pattern produced by light interference
in laser scatter, such that some bistrings resembles a speckled intensity this probability distribution becomes exponentially more difficult as the number of qubits (width) and number of gate cycles (denth) grow.

> So, from what I can tell, they configure their qubits into a pseudo-randomly generated circuit, which, when run. puts the qubits into a state vector that represents a probability distribution over 2^{53} possible states of the qubits. but that distribution is intractable to calculate, or even estimate via sampling using a classical computer simulation. But they sample it by "looking" at the state of the qubits after running the circuit many times.

Isn't this just an example of creating a system whose output is intractable to calculate, and then "calculating" it by simply observing the output of the system?

It sounds similar to saying:

If I spill this pudding cup on the floor, the exact pattern it will form is very chaotic, and intractable for any supercomputer to calculate. But I just invented a new special type of computer: this pudding cup. And I'm going to do the calculation by spilling it on the floor and observing the result. I have achieved pudding supremacy.

What is quantum supremacy?

Quantum supremacy refers to the moment where a functional quantum computer can effectively solve a problem that is not solvable (within decent time frame, $e.g.$ 100 years) with any (super) computer.

This result is a bit biased and overselled: it was obtained using a very specific (ad-hoc) problem that was purposely designed to behave much much better quantumly than classicaly...

What is quantum supremacy?

Quantum supremacy refers to the moment where a functional quantum computer can effectively solve a problem that is not solvable (within decent time frame, $e.g.$ 100 years) with any (super) computer.

This result is a bit biased and overselled: it was obtained using a very specific (ad-hoc) problem that was purposely designed to behave much much better quantumly than classicaly...

It however remains impressive, since no regular computer can do that efficiently. A bit weaker than supremacy is "quantum advantage", where a quantum computer simply performs better than any computer.

Ecole Nationale de l'Aviation Civile

Open challenges towards quantum computing

More work is required to embrace a large scale quantum computer:

developing quantum error-correcting codes for error-free quantum computing

- developing quantum error-correcting codes for error-free quantum computing
- developing reliable quantum memories

- developing quantum error-correcting codes for error-free quantum computing
- developing reliable quantum memories
- quantum measurement (wave function collapsing) is probabilistic

- developing quantum error-correcting codes for error-free quantum computing
- developing reliable quantum memories
- quantum measurement (wave function collapsing) is probabilistic
- **Duilding architectures and interfaces between quantum computers and communication** systems

- developing quantum error-correcting codes for error-free quantum computing
- developing reliable quantum memories
- quantum measurement (wave function collapsing) is probabilistic
- **Duilding architectures and interfaces between quantum computers and communication** systems
- **d** developing quantum programming languages, compilers and middle-ware stack

More work is required to embrace a large scale quantum computer:

- **d** developing quantum error-correcting codes for error-free quantum computing
- developing reliable quantum memories
- quantum measurement (wave function collapsing) is probabilistic
- **Duilding architectures and interfaces between quantum computers and communication** systems
- **d** developing quantum programming languages, compilers and middle-ware stack
- Still, a Sword of Damocles hanging over our heads, and now is the time for designing quantum-safe alternatives.

Outline

- 1 [What you've learnt so far \(should have\)](#page-2-0)
- 2 [Classical vs Quantum computing](#page-13-0)
- [Two noticeable quantum algorithms \(and their impact over cryptography\)](#page-36-0)
- [State-of-the-art quantum computers](#page-77-0)
- [Possible alternatives](#page-117-0)
-

Ecole Nationale de l'Aviation Civile

Possible alternative: physical cryptography

Imagine a lockable-briefcase salesman proposing a "locked-briefcase Internet"

Possible alternative: physical cryptography

- **Imagine a lockable-briefcase salesman proposing a "locked-briefcase Internet"**
- using "provably secure locked-briefcase cryptography":
	- Alice puts secret information into a lockable briefcase.
	- **Alice locks the briefcase.**
	- A courier transports the briefcase from Alice to Bob.
	- Bob unlocks the briefcase and retrieves the information.
	- There is a mathematical proof that the information is hidden!
	- Throw away algorithmic cryptography!

Possible alternative: physical cryptography

- **Imagine a lockable-briefcase salesman proposing a "locked-briefcase Internet"**
- using "provably secure locked-briefcase cryptography":
	- Alice puts secret information into a lockable briefcase.
	- **Alice locks the briefcase.**
	- A courier transports the briefcase from Alice to Bob.
	- Bob unlocks the briefcase and retrieves the information
	- There is a mathematical proof that the information is hidden!
	- Throw away algorithmic cryptography!
- Most common reactions from security experts:
	- **This would make security much worse.**
	- You can't do signatures.
	- **This would be insanely expensive.**
	- We should not dignify this proposal with a response

Security advantages of algorithmic cryptography

Keep secrets heavily shielded inside authorized computers.

Security advantages of algorithmic cryptography

- Keep secrets heavily shielded inside authorized computers.
- \blacksquare Reduce trust in third parties:
	- Reduce reliance on closed-source software and hardware.
	- \blacksquare Increase comprehensiveness of audits.
	- **n** Increase comprehensiveness of formal verification.
	- **Design systems to be secure even if algorithm and public keys are public.**
	- Critical example: signed software updates.

Security advantages of algorithmic cryptography

- Keep secrets heavily shielded inside authorized computers.
- Reduce trust in third parties:
	- Reduce reliance on closed-source software and hardware.
	- \blacksquare Increase comprehensiveness of audits.
	- **n** Increase comprehensiveness of formal verification.
	- **Design systems to be secure even if algorithm and public keys are public.**
	- Critical example: signed software updates.
- **Understand security as thoroughly as possible:**
	- **Publish comprehensive specifications.**
	- Build large research community with clear security goals.
	- **Publicly document attack efforts.**
	- Require systems to convincingly survive many years of analysis.

- **Many stages of research from cryptographic design to deployment:**
	- Explore space of cryptosystems.
	- **Study algorithms for the attackers.**
	- Focus on secure cryptosystems.
	- Study algorithms for the users.
	- Study implementations on real hardware.
	- Study side-channel attacks, fault attacks, etc.
	- Focus on secure, reliable implementations.
	- Focus on implementations meeting performance requirements.
	- Integrate securely into real-world applications.

- **Many stages of research from cryptographic design to deployment:**
	- Explore space of cryptosystems.
	- **Study algorithms for the attackers.**
	- Focus on secure cryptosystems.
	- Study algorithms for the users.
	- Study implementations on real hardware.
	- Study side-channel attacks, fault attacks, etc.
	- Focus on secure, reliable implementations.
	- Focus on implementations meeting performance requirements.
	- \blacksquare Integrate securely into real-world applications.
- **EXample: ECC introduced 1985; big advantages over RSA. Robust ECC started to take** over the Internet in 2015.

- **Many stages of research from cryptographic design to deployment:**
	- Explore space of cryptosystems.
	- **Study algorithms for the attackers.**
	- Focus on secure cryptosystems.
	- Study algorithms for the users.
	- Study implementations on real hardware.
	- Study side-channel attacks, fault attacks, etc.
	- Focus on secure, reliable implementations.
	- **Focus on implementations meeting performance requirements.**
	- \blacksquare Integrate securely into real-world applications.
- Example: ECC introduced 1985; big advantages over RSA. Robust ECC started to take over the Internet in 2015.
- Can't wait for quantum computers before finding a solution!

- **Many stages of research from cryptographic design to deployment:**
	- Explore space of cryptosystems.
	- **Study algorithms for the attackers.**
	- Focus on secure cryptosystems.
	- Study algorithms for the users.
	- Study implementations on real hardware.
	- Study side-channel attacks, fault attacks, etc.
	- Focus on secure, reliable implementations.
	- **Focus on implementations meeting performance requirements.**
	- Integrate securely into real-world applications.
- Example: ECC introduced 1985; big advantages over RSA. Robust ECC started to take over the Internet in 2015.
- Can't wait for quantum computers before finding a solution!

Let's move to post-quantum crypto now!

Outline

- 1 [What you've learnt so far \(should have\)](#page-2-0)
- 2 [Classical vs Quantum computing](#page-13-0)
- [Two noticeable quantum algorithms \(and their impact over cryptography\)](#page-36-0)
- [State-of-the-art quantum computers](#page-77-0)
-
- 6 [Post-quantum cryptography](#page-128-0)

Clarification

What are the alternatives to classical cryptography in presence of an adversary equipped with a large scale quantum computer?

Clarification

What are the alternatives to classical cryptography in presence of an adversary equipped with a large scale quantum computer?

Quantum Key Exchange (out of the scope of this course)

Clarification

What are the alternatives to classical cryptography in presence of an adversary equipped with a large scale quantum computer?

Quantum Key Exchange (out of the scope of this course)

Post-Quantum Cryptography

Ecole Nationale de l'Aviation Civile

Post-Quantum Cryptography

What are the ingredients for building quantum-safe cryptographic primitives?

Lattice-based cryptography

- **Lattice-based cryptography**
- **Exercial** (Error-correcting) Code-based cryptography

- **Lattice-based cryptography**
- **Exercial** (Error-correcting) Code-based cryptography
- Hash (function) based cryptography

- **Lattice-based cryptography**
- **EXECUTE:** (Error-correcting) Code-based cryptography
- Hash (function) based cryptography
- **Multivariate (polynomials)** based cryptography

What are the ingredients for building quantum-safe cryptographic primitives?

- **Lattice-based cryptography**
- **EXECUTE:** (Error-correcting) Code-based cryptography
- **Hash (function)** based cryptography
- **Multivariate (polynomials)** based cryptography
- **In Isogeny (over elliptic curves)** based cryptography

NIST PQC standardization process

NIST National Institute of Standards and Technologies

NIST PQC standardization process

NIST National Institute of Standards and Technologies

- 3rd call for standardization
- Asks for post-quantum cryptographic algorithms
- 3 categories :
	- **Encryption**
	- Key exchange
	- Signature
- **Many candidates:**
	- **Error** correcting codes,
	- **Lattices**.
	- **Multivariate,**
	- **Hash functions,**

 \blacksquare .

NIST PQC standardization process

NUST National Institute of Standards and Technologies

- 3rd call for standardization
- Asks for post-quantum cryptographic algorithms
- 3 categories :
	- **Encryption**
	- Key exchange
	- Signature
- **Many candidates:**
	- **Error** correcting codes,
	- **Lattices**.

 \blacksquare

- **Multivariate,**
- **Hash functions.**
- November 2016: anouncement
- November 2017: submission deadline (82 submissions)
- December 2017: $1st$ round: 69 submissions
- April 2018: $1st$ standardization conference
- January 2019: $2nd$ round: 26 candidates
- March 2019: tweaks for 2nd round
- August 2019: 2^{nd} standardization conference
- July 2020: 3rd round: 7 finalists, 8 alternates
- 2022 \rightarrow 2024: draft standards ready

Hot topic!

Submissions available at:

[https://csrc.nist.gov/Projects/post-quantum-cryptography/](https://csrc.nist.gov/Projects/post-quantum-cryptography/Post-Quantum-Cryptography-Standardization) [Post-Quantum-Cryptography-Standardization](https://csrc.nist.gov/Projects/post-quantum-cryptography/Post-Quantum-Cryptography-Standardization)

source: Dustin Moody, NIST

www.enac.fr

■ <https://www.safecrypto.eu/pqclounge/>

Hot topic!
Below is a timeline of major events with respect to the NIST POC Standardization Process.

source: NIST IR 8240

Hot topic!

Timeline

*This is a tentative timeline, provided for information, and subject to change.

Date

Ecole Nationale de l'Aviation Civile

Outline

6 [Post-quantum cryptography](#page-128-0) **[Lattice-based cryptography](#page-144-0) [Hash-based cryptography](#page--1-0)**

[Code-based cryptography](#page--1-0)

www.enac.fr