

An introduction to Post-Quantum Cryptography (PQC)

Jean-Christophe Deneuville

<jean-christophe.deneuville@enac.fr>

Fall 2020





- **1** What you've learnt so far (should have)
- 2 Classical vs Quantum computing
- **3** Two noticeable quantum algorithms (and their impact over cryptography)
- 4 State-of-the-art quantum computers
- 5 Possible alternatives
- 6 Post-quantum cryptography



www.enac.fr



Outline

- **1** What you've learnt so far (should have)
- 2 Classical vs Quantum computing
- 3 Two noticeable quantum algorithms (and their impact over cryptography)
- State-of-the-art quantum computers
- Possible alternatives
- 6 Post-quantum cryptography



www.enac.fr



What is cryptography





- What is cryptography
- How it relates to information security



- What is cryptography
- How it relates to information security
- Examples of ancestral constructions (Scytale, Cæsar, Vigenere, ...)



- What is cryptography
- How it relates to information security
- Examples of ancestral constructions (Scytale, Cæsar, Vigenere, ...)
- Their weaknesses/cryptanalysis (Al-Kindi (frequency analysis), Babbage, ...)



- What is cryptography
- How it relates to information security
- Examples of ancestral constructions (Scytale, Cæsar, Vigenere, ...)
- Their weaknesses/cryptanalysis (Al-Kindi (frequency analysis), Babbage, ...)
- Recent symmetric constructions (OTP, Enigma, Sigaba, ...)



- What is cryptography
- How it relates to information security
- Examples of ancestral constructions (Scytale, Cæsar, Vigenere, ...)
- Their weaknesses/cryptanalysis (Al-Kindi (frequency analysis), Babbage, ...)
- Recent symmetric constructions (OTP, Enigma, Sigaba, ...)
- Modern symmetric constructions (Stream/Block cipher, DES, AES, Blowfish, RC4, ...)



- What is cryptography
- How it relates to information security
- Examples of ancestral constructions (Scytale, Cæsar, Vigenere, ...)
- Their weaknesses/cryptanalysis (Al-Kindi (frequency analysis), Babbage, ...)
- Recent symmetric constructions (OTP, Enigma, Sigaba, ...)
- Modern symmetric constructions (Stream/Block cipher, DES, AES, Blowfish, RC4, ...)
- Asymmetric cryptography (RSA, Diffie-Hellman, ElGamal, ...)



- What is cryptography
- How it relates to information security
- Examples of ancestral constructions (Scytale, Cæsar, Vigenere, ...)
- Their weaknesses/cryptanalysis (Al-Kindi (frequency analysis), Babbage, ...)
- Recent symmetric constructions (OTP, Enigma, Sigaba, ...)
- Modern symmetric constructions (Stream/Block cipher, DES, AES, Blowfish, RC4, ...)
- Asymmetric cryptography (RSA, Diffie-Hellman, ElGamal, ...)
- (Collision Resistant) Hash Functions (and birthday paradox)



- What is cryptography
- How it relates to information security
- Examples of ancestral constructions (Scytale, Cæsar, Vigenere, ...)
- Their weaknesses/cryptanalysis (Al-Kindi (frequency analysis), Babbage, ...)
- Recent symmetric constructions (OTP, Enigma, Sigaba, ...)
- Modern symmetric constructions (Stream/Block cipher, DES, AES, Blowfish, RC4, ...)
- Asymmetric cryptography (RSA, Diffie-Hellman, ElGamal, ...)
- (Collision Resistant) Hash Functions (and birthday paradox)
- Digital Signatures (RSA, DSA, ECDSA, ...)



- What is cryptography
- How it relates to information security
- Examples of ancestral constructions (Scytale, Cæsar, Vigenere, ...)
- Their weaknesses/cryptanalysis (Al-Kindi (frequency analysis), Babbage, ...)
- Recent symmetric constructions (OTP, Enigma, Sigaba, ...)
- Modern symmetric constructions (Stream/Block cipher, DES, AES, Blowfish, RC4, ...)
- Asymmetric cryptography (RSA, Diffie-Hellman, ElGamal, ...)
- (Collision Resistant) Hash Functions (and birthday paradox)
- Digital Signatures (RSA, DSA, ECDSA, ...)
- Security models





- 2 Classical vs Quantum computing
- **3** Two noticeable quantum algorithms (and their impact over cryptography)
- State-of-the-art quantum computers
- Possible alternatives
- 6 Post-quantum cryptography





Classical computing

A classical computer (Turing machine) processes (through a language) classical boolean circuits.

The quantity of information is measured through Shannon's entropy, data can eventually be compressed, and there exist efficient algorithms for error correction.

Some circuits are computable *i.e.* the machine eventually halts (*e.g.* primality problem), some others aren't (*e.g.* the halting problem).



Current security





Current security vs. classical computing power (2020)

www.enac.fr



1 standard machine: 64 bits architecture $2^{\rm 6}$

www.enac.fr



1 standard machine: 8 cores $2^6\times 2^3$

www.enac.fr



Current security vs. classical computing power (2020)

1 standard machine: 4 GHz $2^6\times 2^3\times 2^2\times 10^9$

www.enac.fr



Current security vs. classical computing power (2020)

1 standard machine: running 1 month $2^6\times2^3\times2^2\times10^9\times60\times60\times24\times30$

www.enac.fr



Current security vs. classical computing power (2020)

 $\begin{array}{l} \mathsf{NSA} \geq 10 \; \mathsf{000} \; \mathsf{standard} \; \mathsf{machines?} \\ 2^6 \times 2^3 \times 2^2 \times 10^9 \times 60 \times 60 \times 24 \times 30 \times 10^4 \end{array}$

www.enac.fr



Current security vs. classical computing power (2020)

NSA \geq 10 000 standard machines?

 $2^6\times2^3\times2^2\times10^9\times60\times60\times24\times30\times10^4\approx2^{80}$ elementary operations



 $\begin{array}{l} \text{NSA} \geq 10 \; \text{000 standard machines?} \quad \mbox{(without possible GPU, ASICS, ...)} \\ 2^6 \times 2^3 \times 2^2 \times 10^9 \times 60 \times 60 \times 24 \times 30 \times 10^4 \approx 2^{80} \; \text{elementary operations} \end{array}$

www.enac.fr



 $\begin{array}{ll} \mathsf{NSA} \geq 10 \; \mathsf{000} \; \mathsf{standard \; machines?} & \text{(without possible GPU, ASICS, ...)} \\ 2^6 \times 2^3 \times 2^2 \times 10^9 \times 60 \times 60 \times 24 \times 30 \times 10^4 \approx 2^{80} \; \mathsf{elementary operations} \end{array}$

A concrete example

During 2018, there were 2^{89} SHA-256 hashes computed on the blockchain BitCoin...



 $\begin{array}{ll} \mathsf{NSA} \geq 10 \; \mathsf{000} \; \mathsf{standard \; machines?} & \text{(without possible GPU, ASICS, ...)} \\ 2^6 \times 2^3 \times 2^2 \times 10^9 \times 60 \times 60 \times 24 \times 30 \times 10^4 \approx 2^{80} \; \mathsf{elementary operations} \end{array}$

A concrete example

During 2018, there were 2^{89} SHA-256 hashes computed on the blockchain BitCoin...

Security in 2020

Setting parameters so that best known attacks have complexity (at least) 2^{128} .



 $\begin{array}{ll} \mathsf{NSA} \geq 10 \; \mathsf{000} \; \mathsf{standard \; machines?} & \text{(without possible GPU, ASICS, ...)} \\ 2^6 \times 2^3 \times 2^2 \times 10^9 \times 60 \times 60 \times 24 \times 30 \times 10^4 \approx 2^{80} \; \mathsf{elementary operations} \end{array}$

A concrete example

During 2018, there were 2^{89} SHA-256 hashes computed on the blockchain BitCoin...

Security in 2020

Setting parameters so that best known attacks have complexity (at least) 2^{128} .

Classical best known attacks:

- Symmetric primitives: brute-force
- Asymmetric primitives: GNFS, sub-exponential complexity



Quantum computing has been suggested by Paul Benioff in 1980. He proposed a quantum mechanical model of the Turing machine, using two ingredients:



Quantum computing has been suggested by Paul Benioff in 1980. He proposed a quantum mechanical model of the Turing machine, using two ingredients:

Superposition: while a bit can be either in a state 0 or 1, a quantum bit (qubit) can be in any superposition of states |0> and |1>.

Quantum computing

Quantum computing has been suggested by Paul Benioff in 1980. He proposed a quantum mechanical model of the Turing machine, using two ingredients:

- Superposition: while a bit can be either in a state 0 or 1, a quantum bit (*qubit*) can be in any *superposition* of states |0⟩ and |1⟩.
- Entanglement: the capability of two qubits to be *correlated*. If Alice and Bob both get one of two entangled qubits, and if Alice measures a |0⟩ at some point, then necessarily Bob must measure the same, as |00⟩ is the only state where Alice's qubit is a |0⟩.

Quantum computing

Quantum computing has been suggested by Paul Benioff in 1980. He proposed a quantum mechanical model of the Turing machine, using two ingredients:

- Superposition: while a bit can be either in a state 0 or 1, a quantum bit (*qubit*) can be in any *superposition* of states |0⟩ and |1⟩.
- Entanglement: the capability of two qubits to be *correlated*. If Alice and Bob both get one of two entangled qubits, and if Alice measures a |0⟩ at some point, then necessarily Bob must measure the same, as |00⟩ is the only state where Alice's qubit is a |0⟩.

Qubits can be "implemented" using the spin of an electron, or the polarization of a photon, ...



As a consequence:

 \blacksquare a vector of n entangled qubits can be in a superposition of any 2^n possible states at the same time,





As a consequence:

- \blacksquare a vector of n entangled qubits can be in a superposition of any 2^n possible states at the same time,
- against 1 among the 2^n possible states for a classical vector of n bits.



As a consequence:

- \blacksquare a vector of n entangled qubits can be in a superposition of any 2^n possible states at the same time,
- against 1 among the 2^n possible states for a classical vector of n bits.
- It is however not possible to observe these states all together at the same time.

Quantum computing

As a consequence:

- \blacksquare a vector of n entangled qubits can be in a superposition of any 2^n possible states at the same time,
- against 1 among the 2^n possible states for a classical vector of n bits.

It is however not possible to observe these states all together at the same time. A quantum algorithm solving a problem needs to make the correct solution (state) exponentially more likely than the other states (cf. quantum annealing / wave function collapsing).


Outline

- 1 What you've learnt so far (should have)
- 2 Classical vs Quantum computing
- **3** Two noticeable quantum algorithms (and their impact over cryptography)
- 4 State-of-the-art quantum computers
- Possible alternatives
- 6 Post-quantum cryptography





Shor's algorithm

SIAM J. COMPUT. Vol. 26, No. 5, pp. 1484–1509, October 1997 C 1997 Society for Industrial and Applied Mathematics 009

POLYNOMIAL-TIME ALGORITHMS FOR PRIME FACTORIZATION AND DISCRETE LOGARITHMS ON A QUANTUM COMPUTER*

PETER W. SHOR[†]

Abstract. A digital computer is generally believed to be an efficient universal computing device; that is, it is believed able to simulate any physical computing device with an increase in computation time by at most a polynomial factor. This may not be true when quantum mechanics is taken into consideration. This paper considers factoring integers and finding discrete logarithms, two problems which are generally thought to be hard on a classical computer and which have been used as the basis of several proposed cryptosystems. Efficient randomized algorithms are given for these two problems on a hypothetical quantum computer. These algorithms take a number of steps polynomial in the input size, e.g., the number of digits of the integer to be factored.



Algorithm 1: ShorAlgorithm(N)

Input: N**Output:** p, q such that N = pq





Algorithm 1: ShorAlgorithm(N)

Input: N Output: p, q such that N = pq1 Pick $g \in \mathbb{Z}_N$ at random;



Algorithm 1: ShorAlgorithm(N)

Input: N Output: p, q such that N = pq1 Pick $g \in \mathbb{Z}_N$ at random; 2 if $gcd(g, N) \neq 1$ then 3 $\ \$ then return (p = gcd(g, N), q = N/p)



Algorithm 1: ShorAlgorithm(N)

Input: N Output: p, q such that N = pq1 Pick $g \in \mathbb{Z}_N$ at random; 2 if $gcd(g, N) \neq 1$ then 3 L then return (p = gcd(g, N), q = N/p)4 Find r such that $g^r \equiv 1[N]$;



Algorithm 1: ShorAlgorithm(N)

Input: N **Output:** p, q such that N = pq1 Pick $q \in \mathbb{Z}_N$ at random: 2 if $gcd(q, N) \neq 1$ then 3 then return (p = gcd(g, N), q = N/p)4 Find r such that $q^r \equiv 1[N]$; 5 if $r \equiv 0[2]$ then 6 return $gcd(q^{r/2} \pm 1, N)$ 7 else 8 go to 1

Ecole Nationale de l'Aviation Civile



Shor's algorithm: how it works

"Find r such that $g^r \equiv 1[N]$;"



Ecole Nationale de l'Aviation Civile



Shor's algorithm: how it works

"Find r such that $g^r \equiv 1[N]$;"

First question: How does finding r such that $g^r \equiv 1[N]$ help factoring?



"Find r such that $g^r \equiv 1[N]$;"

First question: How does finding r such that $g^r \equiv 1[N]$ help factoring?

 $g^r \equiv 1[N] \quad \Leftrightarrow \quad \exists k \in \mathbb{N}^* \text{ such that } g^r = kN+1$





"Find r such that $g^r \equiv 1[N]$;"

First question: How does finding r such that $g^r \equiv 1[N]$ help factoring?

$$\begin{array}{ll} g^r \equiv 1[N] & \Leftrightarrow & \exists k \in \mathbb{N}^* \text{ such that } g^r = kN+1 \\ & \Leftrightarrow & g^r-1 = kN \end{array}$$



"Find r such that $g^r \equiv 1[N]$;"

First question: How does finding r such that $g^r \equiv 1[N]$ help factoring?

$$\begin{split} g^r &\equiv 1[N] &\Leftrightarrow \quad \exists k \in \mathbb{N}^* \text{ such that } g^r = kN + 1 \\ &\Leftrightarrow \quad g^r - 1 = kN \\ &\text{assuming } r \text{ is even}) &\Leftrightarrow \quad \left(g^{r/2} - 1\right) \left(g^{r/2} + 1\right) = kN \end{split}$$



www.enac.fr



"Find r such that $g^r \equiv 1[N]$;"

First question: How does finding r such that $g^r \equiv 1[N]$ help factoring?

$$\begin{split} g^r &\equiv 1[N] &\Leftrightarrow \quad \exists k \in \mathbb{N}^* \text{ such that } g^r = kN + 1 \\ &\Leftrightarrow \quad g^r - 1 = kN \\ \text{assuming } r \text{ is even}) &\Leftrightarrow \quad \left(g^{r/2} - 1\right) \left(g^{r/2} + 1\right) = kN \end{split}$$

Meaning that there is a non-negligible probability that $g^{r/2} \pm 1$ shares non trivial factors with N.

Ecole Nationale de l'Aviation Civile



Shor's algorithm: how it works

Example with N = 314191, find p, q

(source: minutephysics)

Ecole Nationale de l'Aviation Civile



Shor's algorithm: how it works

Example with N = 314191, find p, q

step 1. $g \leftarrow 101$

(source: minutephysics)

www.enac.fr



Example with N = 314191, find p, q

- **step 1**. $g \leftarrow 101$
- **step 2**. $r \leftarrow 4347$

(source: minutephysics)

www.enac.fr



Example with N = 314191, find p, q

- **step 1**. $g \leftarrow 101$
- **step 2**. $r \leftarrow 4347$
- **step 3**. r is **odd**... go to 1

(source: minutephysics)

www.enac.fr



Example with N = 314191, find p, q

- **step 1**. $g \leftarrow 101$
- **step 2**. $r \leftarrow 4347$
- **step 3**. r is **odd**... go to 1
- **step 1**. $g \leftarrow 127$

(source: minutephysics)

www.enac.fr



Example with N = 314191, find p, q

- **step 1**. $g \leftarrow 101$
- **step 2**. $r \leftarrow 4347$
- **step 3**. r is **odd**... go to 1
- step 1. $g \leftarrow 127$
- step 2. $r \leftarrow 17388$

(source: minutephysics)

www.enac.fr



Example with N = 314191, find p, q

- **step 1**. $g \leftarrow 101$
- **step 2**. $r \leftarrow 4347$
- **step 3**. r is **odd**... go to 1
- step 1. $g \leftarrow 127$
- step 2. $r \leftarrow 17388$
- step 3. let us denote $g_p = g^{17388/2} + 1$ and $g_q = g^{17388/2} 1$

(source: minutephysics)



Example with N = 314191, find p, q

- **step 1**. $g \leftarrow 101$
- **step 2**. $r \leftarrow 4347$
- **step 3**. r is **odd**... go to 1
- step 1. $g \leftarrow 127$
- step 2. $r \leftarrow 17388$
- step 3. let us denote $g_p = g^{17388/2} + 1$ and $g_q = g^{17388/2} 1$ we have that $gcd(g_p, N) = 829 =: p$ and $gcd(g_q, N) = 379 =: q$ and indeed, $p \cdot q = 829 \times 379 = 314191 = N$

(source: minutephysics)

www.enac.fr



Second question: "Wait a minute, I was expecting some magic quantum trick out there. Where the is the quantum part?"



Second question: "Wait a minute, I was expecting some magic quantum trick out there. Where the is the quantum part?"

"Find r such that $g^r \equiv 1[N]$;"

www.enac.fr



Second question: "Wait a minute, I was expecting some magic quantum trick out there. Where the is the quantum part?"

"Quantumly find r such that $g^r \equiv 1[N]$;"





"Quantumly find r such that $g^r \equiv 1[N]$;"

The complexity to find the *period* of the function $g \mapsto g^x \mod N$ is:



"Quantumly find r such that $g^r \equiv 1[N]$;"

The complexity to find the *period* of the function $g \mapsto g^x \mod N$ is:

Classically $\mathcal{O}(N)$



"Quantumly find r such that $g^r \equiv 1[N]$;"

The complexity to find the *period* of the function $g \mapsto g^x \mod N$ is:

- Classically $\mathcal{O}(N)$
- Quantumly $\mathcal{O}\left(\log\left(N\right)^3\right)$



"Quantumly find r such that $g^r \equiv 1[N]$;"

The complexity to find the *period* of the function $g \mapsto g^x \mod N$ is:

- Classically $\mathcal{O}(N)$
- Quantumly $\mathcal{O}\left(\log\left(N\right)^{3}\right)$. That's an **exponential** speedup!



Quantum period finding

How does it work? Why is it much much faster quantumly?

Fourier Transform is THE tool to analyse frequencies. Fortunately, it has a quantum equivalent: QFT.

Quantum computing allows to provide QFT a superposition of every possible states (assuming enough qubits).



• Factoring becomes polynomial-time $\mathcal{O}\left(\left(\log N\right)^2 \left(\log \log N\right) \left(\log \log \log N\right)\right)$



• Factoring becomes polynomial-time $\mathcal{O}\left(\left(\log N\right)^2 \left(\log \log N\right) \left(\log \log \log N\right)\right)$ against $\exp\left(1.9(\log N)^{1/3} (\log \log N)^{2/3}\right)$ classically

www.enac.fr



• Factoring becomes polynomial-time $\mathcal{O}\left((\log N)^2 (\log \log N) (\log \log \log N)\right)$ against $\exp\left(1.9(\log N)^{1/3} (\log \log N)^{2/3}\right)$ classically





- Factoring becomes polynomial-time $\mathcal{O}\left((\log N)^2 (\log \log N) (\log \log \log N)\right)$ against $\exp\left(1.9(\log N)^{1/3} (\log \log N)^{2/3}\right)$ classically
- Discrete logarithm becomes almost polynomial-time





- Factoring becomes polynomial-time $\mathcal{O}\left(\left(\log N\right)^2 \left(\log \log N\right) \left(\log \log \log N\right)\right)$ against exp $\left(1.9(\log N)^{1/3} (\log \log N)^{2/3}\right)$ classically
- Discrete logarithm becomes almost polynomial-time

No more RSA, DSA, ECDSA, ElGamal, ...





- Factoring becomes polynomial-time $\mathcal{O}\left((\log N)^2 (\log \log N) (\log \log \log N)\right)$ against $\exp\left(1.9(\log N)^{1/3} (\log \log N)^{2/3}\right)$ classically
- Discrete logarithm becomes almost polynomial-time

```
No more RSA, DSA, ECDSA, ElGamal, ...
```



In other words, security as we know it collapses...



Grover's algorithm

A fast quantum mechanical algorithm for database search

Lov K. Grover 3C-404A, Bell Labs 600 Mountain Avenue Murray Hill NJ 07974 *lkgrover@bell-labs.com*

Summary

Imagine a phone directory containing N names arranged in completely random order. In order to find someone's phone number with a probability of $\frac{1}{2}$, any classical algorithm (whether deterministic or probabilistic) will need to look at a minimum of $\frac{N}{2}$ names. Quantum mechanical systems can be in a superposition of states and simultaneously examine multiple names. By properly adjusting the phases of various operations, successful computations reinforce each other while others interfere randomly. As a result, the desired phone number can be obtained in only $O(\sqrt{N})$ steps. The algorithm is within a small constant factor of the fastest possible quantum mechanical algorithm.


(*n*-entries unsorted) Database search takes $\mathcal{O}(\sqrt{n})$ queries instead of $\mathcal{O}(n)$.



(*n*-entries unsorted) Database search takes $\mathcal{O}(\sqrt{n})$ queries instead of $\mathcal{O}(n)$.

Consequence over symmetric crypto:





(*n*-entries unsorted) Database search takes $\mathcal{O}(\sqrt{n})$ queries instead of $\mathcal{O}(n)$.

Consequence over symmetric crypto:

ightarrow The length of the secret key must be **doubled** to preserve the same level of security



(*n*-entries unsorted) Database search takes $\mathcal{O}(\sqrt{n})$ queries instead of $\mathcal{O}(n)$.

Consequence over symmetric crypto:

ightarrow The length of the secret key must be **doubled** to preserve the same level of security

Consequence over hash functions:



(*n*-entries unsorted) Database search takes $\mathcal{O}(\sqrt{n})$ queries instead of $\mathcal{O}(n)$.

Consequence over symmetric crypto:

 $\rightarrow\,$ The length of the secret key must be doubled to preserve the same level of security

Consequence over hash functions:

 \rightarrow More tricky (depending on the model, the size of the quantum computer, …), at least +33% to preserve the security level



Outline

- 1 What you've learnt so far (should have)
- 2 Classical vs Quantum computing
- 3 Two noticeable quantum algorithms (and their impact over cryptography)
- 4 State-of-the-art quantum computers
- 5 Possible alternatives
- 6 Post-quantum cryptography





How far are we from a large-scale quantum computer?

10,000



A quantum analog to Moore's law: the number of qubits (y-axe) approximately doubles every year (x-axe). (Source: D-Waye)

Ecole Nationale de l'Aviation Civile



Large-scale quantum computing: a caveat

This analog to Moore's law has several drawbacks:

www.enac.fr



This analog to Moore's law has several drawbacks:

essentially corresponds to multiple 32 qubits architectures mounted in parallel



This analog to Moore's law has several drawbacks:

- essentially corresponds to multiple 32 qubits architectures mounted in parallel
- fault-tolerance remains an open problem



This analog to Moore's law has several drawbacks:

- essentially corresponds to multiple 32 qubits architectures mounted in parallel
- fault-tolerance remains an open problem
- still far from what is required to factor 2048 bits moduli



This analog to Moore's law has several drawbacks:

- essentially corresponds to multiple 32 qubits architectures mounted in parallel
- fault-tolerance remains an open problem
- still far from what is required to factor 2048 bits moduli

In 2020, the largest quantum computer features 72 qubits (Google).

Ecole Nationale de l'Aviation Civile



Some perspective A bit of perspective regarding quantum stuffs.

(full story here)

www.enac.fr



Some perspective A bit of perspective regarding quantum stuffs.

1968: Wiesner describes conjugate (quantum) coding

(full story here)

www.enac.fr



Some perspective A bit of perspective regarding quantum stuffs.

- 1968: Wiesner describes conjugate (quantum) coding
- 1984: Bennett & Brassard use Wiesner's coding for QKD

(full story here)

www.enac.fr



Some perspective A bit of perspective regarding quantum stuffs.

- 1968: Wiesner describes conjugate (quantum) coding
- 1984: Bennett & Brassard use Wiesner's coding for QKD
- 1994: Shor's algorithm

(full story here)

www.enac.fr



A bit of perspective regarding quantum stuffs.

(full story here)

- 1968: Wiesner describes conjugate (quantum) coding
- 1984: Bennett & Brassard use Wiesner's coding for QKD
- 1994: Shor's algorithm
- 1995: 1st US DoD workshop on Quantum Crypto / Shor proposes quantum error correction





A bit of perspective regarding quantum stuffs.

(full story here)

- 1968: Wiesner describes conjugate (quantum) coding
- 1984: Bennett & Brassard use Wiesner's coding for QKD
- 1994: Shor's algorithm
- 1995: 1st US DoD workshop on Quantum Crypto / Shor proposes quantum error correction
- 1996: Grover's algorithm





- A bit of perspective regarding quantum stuffs.
- 1968: Wiesner describes conjugate (quantum) coding
- 1984: Bennett & Brassard use Wiesner's coding for QKD
- 1994: Shor's algorithm
- 1995: 1st US DoD workshop on Quantum Crypto / Shor proposes quantum error correction
- 1996: Grover's algorithm
- 2001: First execution of Shor's algorithm, factoring 15

(full story here)

www.enac.fr



- A bit of perspective regarding quantum stuffs.
- 1968: Wiesner describes conjugate (quantum) coding
- 1984: Bennett & Brassard use Wiesner's coding for QKD
- 1994: Shor's algorithm
- 1995: 1st US DoD workshop on Quantum Crypto / Shor proposes quantum error correction
- 1996: Grover's algorithm
- 2001: First execution of Shor's algorithm, factoring 15
- 2002: Creation of the Institute for Quantum Computing

(full story here)



- A bit of perspective regarding quantum stuffs.
- 1968: Wiesner describes conjugate (quantum) coding
- 1984: Bennett & Brassard use Wiesner's coding for QKD
- 1994: Shor's algorithm
- 1995: 1st US DoD workshop on Quantum Crypto / Shor proposes quantum error correction
- 1996: Grover's algorithm
- 2001: First execution of Shor's algorithm, factoring 15
- 2002: Creation of the Institute for Quantum Computing
- 2003: DARPA's quantum network operational

(full story here)



- A bit of perspective regarding quantum stuffs.
- 1968: Wiesner describes conjugate (quantum) coding
- 1984: Bennett & Brassard use Wiesner's coding for QKD
- 1994: Shor's algorithm
- 1995: 1st US DoD workshop on Quantum Crypto / Shor proposes quantum error correction
- 1996: Grover's algorithm
- 2001: First execution of Shor's algorithm, factoring 15
- 2002: Creation of the Institute for Quantum Computing
- 2003: DARPA's quantum network operational
- 2009: qubits lifetime $\simeq 100$ ms

(full story here)



- A bit of perspective regarding quantum stuffs.
- 1968: Wiesner describes conjugate (quantum) coding
- 1984: Bennett & Brassard use Wiesner's coding for QKD
- 1994: Shor's algorithm
- 1995: 1st US DoD workshop on Quantum Crypto / Shor proposes quantum error correction
- 1996: Grover's algorithm
- 2001: First execution of Shor's algorithm, factoring 15
- 2002: Creation of the Institute for Quantum Computing
- 2003: DARPA's quantum network operational
- **2009**: qubits lifetime $\simeq 100$ ms
- 2012: D-Wave claims a quantum computation using 84 qubits (24 computational)

(full story here)



- A bit of perspective regarding quantum stuffs.
- 1968: Wiesner describes conjugate (quantum) coding
- 1984: Bennett & Brassard use Wiesner's coding for QKD
- 1994: Shor's algorithm
- 1995: 1st US DoD workshop on Quantum Crypto / Shor proposes quantum error correction
- 1996: Grover's algorithm
- 2001: First execution of Shor's algorithm, factoring 15
- 2002: Creation of the Institute for Quantum Computing
- 2003: DARPA's quantum network operational
- 2009: qubits lifetime $\simeq 100~{
 m ms}$
- 2012: D-Wave claims a quantum computation using 84 qubits (24 computational)
- 2013: coherence time 39mins at room temperature / 1st Snowden revelations

(full story here)



- A bit of perspective regarding quantum stuffs.
- 1968: Wiesner describes conjugate (quantum) coding
- 1984: Bennett & Brassard use Wiesner's coding for QKD
- 1994: Shor's algorithm
- 1995: 1st US DoD workshop on Quantum Crypto / Shor proposes quantum error correction
- 1996: Grover's algorithm
- 2001: First execution of Shor's algorithm, factoring 15
- 2002: Creation of the Institute for Quantum Computing
- 2003: DARPA's quantum network operational
- 2009: qubits lifetime $\simeq 100~{
 m ms}$
- 2012: D-Wave claims a quantum computation using 84 qubits (24 computational)
- 2013: coherence time 39mins at room temperature / 1st Snowden revelations
- 2014: NSA project "Penetrating Hard Target" aims at breaking quantumly strong crypto

(full story here)



- A bit of perspective regarding quantum stuffs.
- 1968: Wiesner describes conjugate (quantum) coding
- 1984: Bennett & Brassard use Wiesner's coding for QKD
- 1994: Shor's algorithm
- 1995: 1st US DoD workshop on Quantum Crypto / Shor proposes quantum error correction
- 1996: Grover's algorithm
- 2001: First execution of Shor's algorithm, factoring 15
- 2002: Creation of the Institute for Quantum Computing
- 2003: DARPA's quantum network operational
- **2009**: qubits lifetime $\simeq 100 \text{ ms}$
- 2012: D-Wave claims a quantum computation using 84 qubits (24 computational)
- 2013: coherence time 39mins at room temperature / 1st Snowden revelations
- 2014: NSA project "Penetrating Hard Target" aims at breaking quantumly strong crypto
- 2015: NSA statement / 2016: NIST preparing PQC standardization

(full story here)

www.enac.fr



- A bit of perspective regarding quantum stuffs.
- 1968: Wiesner describes conjugate (quantum) coding
- 1984: Bennett & Brassard use Wiesner's coding for QKD
- 1994: Shor's algorithm
- 1995: 1st US DoD workshop on Quantum Crypto / Shor proposes quantum error correction
- 1996: Grover's algorithm
- 2001: First execution of Shor's algorithm, factoring 15
- 2002: Creation of the Institute for Quantum Computing
- 2003: DARPA's quantum network operational
- **2009**: qubits lifetime $\simeq 100 \text{ ms}$
- 2012: D-Wave claims a quantum computation using 84 qubits (24 computational)
- 2013: coherence time 39mins at room temperature / 1st Snowden revelations
- 2014: NSA project "Penetrating Hard Target" aims at breaking quantumly strong crypto
- 2015: NSA statement / 2016: NIST preparing PQC standardization
- 2017: industry race for largest quantum computer / D-Wave 2000Q / NIST PQC starts

(full story here)



- A bit of perspective regarding quantum stuffs.
- 1968: Wiesner describes conjugate (quantum) coding
- 1984: Bennett & Brassard use Wiesner's coding for QKD
- 1994: Shor's algorithm
- 1995: 1st US DoD workshop on Quantum Crypto / Shor proposes quantum error correction
- 1996: Grover's algorithm
- 2001: First execution of Shor's algorithm, factoring 15
- 2002: Creation of the Institute for Quantum Computing
- 2003: DARPA's quantum network operational
- **2009**: qubits lifetime $\simeq 100 \text{ ms}$
- 2012: D-Wave claims a quantum computation using 84 qubits (24 computational)
- 2013: coherence time 39mins at room temperature / 1st Snowden revelations
- 2014: NSA project "Penetrating Hard Target" aims at breaking quantumly strong crypto
- 2015: NSA statement / 2016: NIST preparing PQC standardization
- 2017: industry race for largest quantum computer / D-Wave 2000Q / NIST PQC starts
- 2018: Google announces a 72-qubit quantum chip / 2019: quantum supremacy

24

www.enac.f

(full story here)

Ecole Nationale de l'Aviation Civile



Hot news!



Ecole Nationale de l'Aviation Civile Google Claims Quantum Supremacy, Marking a Hot news! www.enac.fr

Ecole Nationale de l'Aviation Civile Hot news! Google's supposed milestone achievement became public last month when a Google Claims' Quantum Supremacy' Marking ÊNĂĊ preprint scientific paper accidentally leaked on the website of NASA, a collaborator, as Fortune reported at the time. Google has said nothing about the potentially historic experiment since then, lending credence to whispers that its researchers are bound to silence under the terms of a news embargo by a major science journal, unable to disclose more information until a certain date which is presumed to be www.enac.fr



Post-quantum cryptography / State-of-the-art quantum computers

Ecole Nationale de l'Aviation Civile



What is quantum supremacy?



What is quantum supremacy?

Quantum supremacy refers to the moment where a functional quantum computer can effectively solve a problem that is not solvable (within decent time frame, *e.g.* 100 years) with any (super) computer.

129

-

*



Quantum supre effectively solve any (super) cor

Wha

Why is Google's quantum supremacy experiment impressive?

Asked 13 days ago Active 11 days ago Viewed 12k times

In the Nature paper published by Google, they say,

To demonstrate quantum supremacy, we compare our quantum processor against state-of-the-art classical compaters in the task of sampling the output of a pseudo-random quantum circuit. Random circuits are a suitable choice for benchmarking because they do not possess structure and therefore allow for limited guarantees of computational hardness. We design the circuits to entangle as set of quantum bis (qubits) by reposted application of single-qubit and two-qubit logical operations. Sampling the quantum circuit's output produces a set of bistrings, for example (0000101, 101100, ...). Owing to quantum interference, the probability distribution of the bistrings resembles a speckled intensity pattern produced by light interference in laser scatter, such that some bistrings are mark more likely to occur than others. Classically computing this probability distribution becomes exponentially more difficult as the number of qubits (width) and number of guze cycles (depth) grow.

So, from what I can tell, they configure their qubits into a pseudo-randomly generated circuit, which, when run, puts the qubits into a state vector that represents a probability distribution over 2^{20} possible states of the qubits, but that distribution is intractable to calculate, or even estimate via sampling using a classical computer simulation. But they sample it by 'looking' at the state of the qubits after running the circuit many times.

Isn't this just an example of creating a system whose output is intractable to calculate, and then "calculating" it by simply observing the output of the system?

It sounds similar to saying:

If I spill this pudding cup on the floor, the exact pattern it will form is very chaotic, and intractable for any supercomputer to calculate. But I just invented a new special type of computer, this pudding cup. And I'm going to do the calculation by spilling it on the floor and observing the result. I have achieved pudding supremacy. ntum computer can frame, *e.g.* 100 years) with



What why is Google's quantum supremacy experiment impressive?

Asked 13 days ago Active 11 days ago Viewed 12k times

In the Nature paper published by Google, they say,

Quantum suprem weffectively solve a many (super) com

To demonstrate quantum supremacy, we compare our quantum processor against stane-of-the-art classical computers in the task of sampling the output of a pseudo-random quantum circuit. Random circuits are a suitable choice for benchmarking because they do not possess structure and therefore allow for limited guarantees of computational hardness. We design the circuits to entangle as est of quantum bits (qubits) by repeated application of single-qubit and two-qubit logical operations. Sampling the quantum circuit's output produces as est of bistrings, for exemple (0000101), 01100, ...). Oving to quantum interference, the probability distribution of the bistrings are much more likely to occur than others. Classically computing this probability distribution becomes exponentially more difficult as the number of qubits (width) and number of gate cycles (depth) grov.

So, from what I can tell, they configure their qubits into a pseudo-randomly generated circuit, which, when run, puts the qubits into a state vector that represents a probability distribution over 2³⁰ possible states of the qubits, but that distribution is intractable to calculate, or even estimate via sampling using a classical computer simulation. But they sample it by Tooking" at the state of the qubits after running the circuit many times.

Isn't this just an example of creating a system whose output is intractable to calculate, and then "calculating" it by simply observing the output of the system?

It sounds similar to saying:

If I spill this pudding cup on the floor, the exact pattern it will form is very chaotic, and intractable for any supercomputer to calculate. But I just invented a new special type of computer: this pudding cup. And I'm going to do the calculation by spilling it on the floor and observing the result. I have achieved pudding supremacy.

antum computer can ie frame, *e.g.* 100 years) with


What is quantum supremacy?

Quantum supremacy refers to the moment where a functional quantum computer can effectively solve a problem that is not solvable (within decent time frame, *e.g.* 100 years) with any (super) computer.

This result is a bit biased and overselled: it was obtained using a very specific (ad-hoc) problem that was purposely designed to behave much much better quantumly than classicaly...



What is quantum supremacy?

Quantum supremacy refers to the moment where a functional quantum computer can effectively solve a problem that is not solvable (within decent time frame, *e.g.* 100 years) with any (super) computer.

This result is a bit biased and overselled: it was obtained using a very specific (ad-hoc) problem that was purposely designed to behave much much better quantumly than classicaly...

It however remains impressive, since no regular computer can do that efficiently. A bit weaker than supremacy is "quantum advantage", where a quantum computer simply performs better than any computer.

Ecole Nationale de l'Aviation Civile



Open challenges towards quantum computing





More work is required to embrace a large scale quantum computer:

developing quantum error-correcting codes for error-free quantum computing



- developing quantum error-correcting codes for error-free quantum computing
- developing reliable quantum memories



- developing quantum error-correcting codes for error-free quantum computing
- developing reliable quantum memories
- quantum measurement (wave function collapsing) is probabilistic



- developing quantum error-correcting codes for error-free quantum computing
- developing reliable quantum memories
- quantum measurement (wave function collapsing) is probabilistic
- building architectures and interfaces between quantum computers and communication systems



- developing quantum error-correcting codes for error-free quantum computing
- developing reliable quantum memories
- quantum measurement (wave function collapsing) is probabilistic
- building architectures and interfaces between quantum computers and communication systems
- developing quantum programming languages, compilers and middle-ware stack



More work is required to embrace a large scale quantum computer:

- developing quantum error-correcting codes for error-free quantum computing
- developing reliable quantum memories
- quantum measurement (wave function collapsing) is probabilistic
- building architectures and interfaces between quantum computers and communication systems
- developing quantum programming languages, compilers and middle-ware stack

Still, a Sword of Damocles hanging over our heads, and **now** is the time for designing **quantum-safe** alternatives.



Outline

- 1 What you've learnt so far (should have)
- 2 Classical vs Quantum computing
- 3 Two noticeable quantum algorithms (and their impact over cryptography)
- 4 State-of-the-art quantum computers
- 5 Possible alternatives
- 6 Post-quantum cryptography



www.enac.fr

Ecole Nationale de l'Aviation Civile



Possible alternative: physical cryptography

Imagine a lockable-briefcase salesman proposing a "locked-briefcase Internet"



www.enac.fr



Possible alternative: physical cryptography

- Imagine a lockable-briefcase salesman proposing a "locked-briefcase Internet"
- using "provably secure locked-briefcase cryptography":
 - Alice puts secret information into a lockable briefcase.
 - Alice locks the briefcase.
 - A courier transports the briefcase from Alice to Bob.
 - Bob unlocks the briefcase and retrieves the information.
 - There is a mathematical proof that the information is hidden!
 - Throw away algorithmic cryptography!





Possible alternative: physical cryptography

- Imagine a lockable-briefcase salesman proposing a "locked-briefcase Internet"
- using "provably secure locked-briefcase cryptography":
 - Alice puts secret information into a lockable briefcase.
 - Alice locks the briefcase.
 - A courier transports the briefcase from Alice to Bob.
 - Bob unlocks the briefcase and retrieves the information.
 - There is a mathematical proof that the information is hidden!
 - Throw away algorithmic cryptography!
- Most common reactions from security experts:
 - This would make security much worse.
 - You can't do signatures.
 - This would be insanely expensive.
 - We should not dignify this proposal with a response







Security advantages of algorithmic cryptography

• Keep secrets heavily shielded inside authorized computers.



Security advantages of algorithmic cryptography

- Keep secrets heavily shielded inside authorized computers.
- Reduce trust in third parties:
 - Reduce reliance on closed-source software and hardware.
 - Increase comprehensiveness of audits.
 - Increase comprehensiveness of formal verification.
 - Design systems to be secure even if algorithm and public keys are public.
 - Critical example: signed software updates.



Security advantages of algorithmic cryptography

- Keep secrets heavily shielded inside authorized computers.
- Reduce trust in third parties:
 - Reduce reliance on closed-source software and hardware.
 - Increase comprehensiveness of audits.
 - Increase comprehensiveness of formal verification.
 - Design systems to be secure even if algorithm and public keys are public.
 - Critical example: signed software updates.
- Understand security as thoroughly as possible:
 - Publish comprehensive specifications.
 - Build large research community with clear security goals.
 - Publicly document attack efforts.
 - Require systems to convincingly survive many years of analysis.



- Many stages of research from cryptographic design to deployment:
 - Explore space of cryptosystems.
 - Study algorithms for the attackers.
 - Focus on secure cryptosystems.
 - Study algorithms for the users.
 - Study implementations on real hardware.
 - Study side-channel attacks, fault attacks, etc.
 - Focus on secure, reliable implementations.
 - Focus on implementations meeting performance requirements.
 - Integrate securely into real-world applications.



- Many stages of research from cryptographic design to deployment:
 - Explore space of cryptosystems.
 - Study algorithms for the attackers.
 - Focus on secure cryptosystems.
 - Study algorithms for the users.
 - Study implementations on real hardware.
 - Study side-channel attacks, fault attacks, etc.
 - Focus on secure, reliable implementations.
 - Focus on implementations meeting performance requirements.
 - Integrate securely into real-world applications.
- Example: ECC introduced 1985; big advantages over RSA. Robust ECC started to take over the Internet in 2015.



- Many stages of research from cryptographic design to deployment:
 - Explore space of cryptosystems.
 - Study algorithms for the attackers.
 - Focus on secure cryptosystems.
 - Study algorithms for the users.
 - Study implementations on real hardware.
 - Study side-channel attacks, fault attacks, etc.
 - Focus on secure, reliable implementations.
 - Focus on implementations meeting performance requirements.
 - Integrate securely into real-world applications.
- Example: ECC introduced 1985; big advantages over RSA. Robust ECC started to take over the Internet in 2015.
- Can't wait for quantum computers before finding a solution!



- Many stages of research from cryptographic design to deployment:
 - Explore space of cryptosystems.
 - Study algorithms for the attackers.
 - Focus on secure cryptosystems.
 - Study algorithms for the users.
 - Study implementations on real hardware.
 - Study side-channel attacks, fault attacks, etc.
 - Focus on secure, reliable implementations.
 - Focus on implementations meeting performance requirements.
 - Integrate securely into real-world applications.
- Example: ECC introduced 1985; big advantages over RSA. Robust ECC started to take over the Internet in 2015.
- Can't wait for quantum computers before finding a solution!

Let's move to post-quantum crypto now!



Outline

- 1 What you've learnt so far (should have)
- 2 Classical vs Quantum computing
- 3 Two noticeable quantum algorithms (and their impact over cryptography)
- State-of-the-art quantum computers
- Possible alternatives
- 6 Post-quantum cryptography



www.enac.fr



Clarification

What are the alternatives to classical cryptography in presence of an adversary equipped with a large scale quantum computer?

www.enac.fr



Clarification

What are the alternatives to classical cryptography in presence of an adversary equipped with a large scale quantum computer?

Quantum Key Exchange (out of the scope of this course)





Clarification

What are the alternatives to classical cryptography in presence of an adversary equipped with a large scale quantum computer?

Quantum Key Exchange (out of the scope of this course)



Post-Quantum Cryptography



Ecole Nationale de l'Aviation Civile



Post-Quantum Cryptography

What are the ingredients for building quantum-safe cryptographic primitives?





What are the ingredients for building quantum-safe cryptographic primitives?

Lattice-based cryptography



What are the ingredients for building quantum-safe cryptographic primitives?

- Lattice-based cryptography
- (Error-correcting) Code-based cryptography





What are the ingredients for building quantum-safe cryptographic primitives?

- Lattice-based cryptography
- (Error-correcting) Code-based cryptography
- Hash (function) based cryptography



What are the ingredients for building quantum-safe cryptographic primitives?

- Lattice-based cryptography
- (Error-correcting) Code-based cryptography
- Hash (function) based cryptography
- Multivariate (polynomials) based cryptography



What are the ingredients for building quantum-safe cryptographic primitives?

- Lattice-based cryptography
- (Error-correcting) Code-based cryptography
- Hash (function) based cryptography
- Multivariate (polynomials) based cryptography
- Isogeny (over elliptic curves) based cryptography



NIST PQC standardization process

NIST National Institute of Standards and Technologies



www.enac.fr



NIST PQC standardization process

NST National Institute of Standards and Technologies

- 3rd call for standardization
- Asks for post-quantum cryptographic algorithms
- 3 categories :
 - Encryption
 - Key exchange
 - Signature
- Many candidates:
 - Error correcting codes,
 - Lattices,
 - Multivariate,
 - Hash functions,

...

www.enac.fr



NIST PQC standardization process

NST National Institute of Standards and Technologies

- 3rd call for standardization
- Asks for post-quantum cryptographic algorithms
- 3 categories :
 - Encryption
 - Key exchange
 - Signature
- Many candidates:
 - Error correcting codes,
 - Lattices,
 - Multivariate,
 - Hash functions,

- November 2016: anouncement
- November 2017: submission deadline (82 submissions)
- December 2017: 1st round: 69 submissions
- April 2018: 1st standardization conference
- January 2019: 2nd round: 26 candidates
- March 2019: tweaks for 2nd round
- August 2019: 2nd standardization conference
- July 2020: 3rd round: 7 finalists, 8 alternates
- \blacksquare 2022 \rightarrow 2024: draft standards ready

· · · ·



Hot topic!

	Signatures	KEM/Encryption	Overall
Lattice-based	4	24	28
Code-based	5	19	24
Multi-variate	7	6	13
Hash-based	4		4
Other	3	10	13
Total	23	59	82

Submissions available at:

https://csrc.nist.gov/Projects/post-quantum-cryptography/ Post-Quantum-Cryptography-Standardization

source: Dustin Moody, NIST

https://www.safecrypto.eu/pqclounge/



Hot topic!

Below is a timeline of major events with respect to the NIST PQC Standardization Process.

•	April 2-3, 2015	Workshop on Cybersecurity in a Post-Quantum World, NIST, Gaithersburg, MD
•	February 24, 2016	PQC Standardization: Announcement and outline of NIST's Call for Submissions presentation given at PQCrypto 2016
•	April 28, 2016	NISTIR 8105, Report on Post-Quantum Cryptography, released
•	August 2, 2016	Federal Register Notice - Proposed Requirements and
		Evaluation Criteria announced for public comment
•	December 20, 2016	Federal Register Notice – Announcing Request for Nomination for Public-Key Post-Ouantum Cryptographic Algorithms
•	November 30, 2017	Submission Deadline for NIST PQC Standardization Process
•	December 20, 2017	First-Round Candidates were announced. The public comment period on the first-round candidates began.
•	April 11-13, 2018	First NIST PQC Standardization Conference, Ft. Lauderdale, FL
•	January 30, 2019	The First Round ended and the Second Round began. Second- Round candidates announced. The public comment period on the second-round candidates began.
•	March 15, 2019	Deadline for updated submission packages for the Second Round
•	August 22-24, 2019	2 nd NIST POC Standardization Conference, Santa Barbara, CA



source: NIST IR 8240

www.enac.fr



Hot topic!

Timeline

*This is a tentative timeline, provided for information, and subject to change.

Date

Feb 24-26, 2016	NIST Presentation at PQCrypto 2016: <u>Announcement and outline of NIST's Call for Submissions (Fall</u> 2016), Dustin Moody
April 28, 2016	NIST releases NISTIR 8105, Report on Post-Quantum Cryptography
Dec 20, 2016	Formal Call for Proposals
Nov 30, 2017	Deadline for submissions
Dec 4, 2017	NIST Presentation at AsiaCrypt 2017: <u>The Ship Has Sailed: The NIST Post-Quantum Crypto "Competition"</u> , Dustin Moody
Dec 21, 2017	Round 1 algorithms announced (69 submissions accepted as "complete and proper")
Apr 11, 2018	NIST Presentation at PQCrypto 2018: <u>Let's Get Ready to Rumble - The NIST PQC "Competition"</u> , Dustin Moody
April 11-13, 2018	First PQC Standardization Conference - Submitter's Presentations
January 30, 2019	Second Round Candidates announced (26 algorithms)
March 15, 2019	Deadline for updated submission packages for the Second Round
May 8-10, 2019	NIST Presentation at PQCrypto 2019: <u>Round 2 of the NIST PQC "Competition" - What was NIST</u> <u>Thinking</u> ? (Spring 2019), <i>Dustin Moody</i>
August 22-24, 2019	Second PQC Standardization Conference
2020/2021	Round 3 begins or select algorithms
2022/2024	Draft Standards Available


Ecole Nationale de l'Aviation Civile



Outline



6 Post-quantum cryptography Lattice-based cryptography

Hash-based cryptographyCode-based cryptography

www.enac.fr

39