An introduction to Post-Quantum Cryptography (PQC)

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Fall 2019

TLS-SEC

Outline

1. What you've learnt so far (should have)
2. Classical vs Quantum computing
3. Two noticeable quantum algorithms (and their impact over cryptography)
4. State-of-the-art quantum computers
5. Quantum safe alternatives

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- Security models

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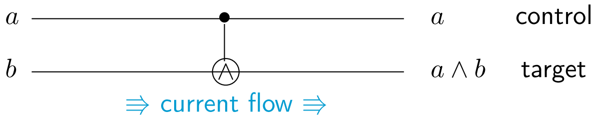
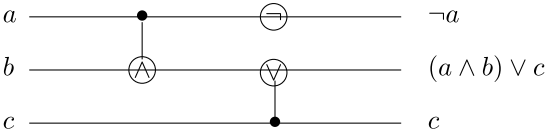
Classical Boolean Circuits

source: J. Royer

We view them as naming maps $\{0,1\}^n \to \{0,1\}^n$



Now consider



We can describe this by either of:

- $b \leftarrow a \wedge b; \quad a \leftarrow \neg a; \quad b \leftarrow b \vee c$
- $|a, b, c\rangle \;\mapsto\; |a, a \wedge b, c\rangle \;\mapsto\; |\neg a, a \wedge b, c\rangle \;\mapsto\; |\neg a, (a \wedge b) \vee c, c\rangle$

$|x, y, z\rangle$ = *state vector*

Classical computing

A classical computer (Turing machine) processes (through a language) classical boolean circuits.

The quantity of information is measured through Shannon's entropy, data can eventually be compressed, and there exist efficient algorithms for error correction.

Some circuits are computable *i.e.* the machine eventually halts (*e.g.* primality problem), some others aren't (*e.g.* the halting problem).

Current security

Current security vs. classical computing power (2019)

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1 standard machine: 64 bits architecture
2^6

Current security vs. classical computing power (2019)

1 standard machine: 8 cores

$2^6 \times 2^4$

Current security vs. classical computing power (2019)

1 standard machine: 4 GHz

$2^6 \times 2^4 \times 2^2 \times 10^9$

Current security vs. classical computing power (2019)

1 standard machine: running 1 month

$2^6 \times 2^4 \times 2^2 \times 10^9 \times 60 \times 60 \times 24 \times 30$

Current security vs. classical computing power (2019)

NSA $\geq$ 10 000 standard machines?

$$2^6 \times 2^4 \times 2^2 \times 10^9 \times 60 \times 60 \times 24 \times 30 \times 10^5$$

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A concrete example

During 2018, there were 2^{89} SHA-256 hashes computed on the blockchain BitCoin...

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Security in 2019

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Classical best known attacks:

- Symmetric primitives: brute-force
- Asymmetric primitives: GNFS, sub-exponential complexity

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- Superposition: while a bit can be either in a state 0 or 1, a quantum bit (*qubit*) can be in any *superposition* of states $|0\rangle$ and $|1\rangle$.
- Entanglement: the capability of two qubits to be *correlated*. If Alice and Bob both get one of two entangled qubits, and if Alice measures a $|0\rangle$ at some point, then necessarily Bob must measure the same, as $|00\rangle$ is the only state where Alice's qubit is a $|0\rangle$.

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Qubits can be "implemented" using the spin of an electron, or the polarization of a photon, ...

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Shor's algorithm

SIAM J. COMPUT.
Vol. 26, No. 5, pp. 1484–1509, October 1997
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009

POLYNOMIAL-TIME ALGORITHMS FOR PRIME FACTORIZATION AND DISCRETE LOGARITHMS ON A QUANTUM COMPUTER*

PETER W. SHOR†

Abstract. A digital computer is generally believed to be an efficient universal computing device; that is, it is believed able to simulate any physical computing device with an increase in computation time by at most a polynomial factor. This may not be true when quantum mechanics is taken into consideration. This paper considers factoring integers and finding discrete logarithms, two problems which are generally thought to be hard on a classical computer and which have been used as the basis of several proposed cryptosystems. Efficient randomized algorithms are given for these two problems on a hypothetical quantum computer. These algorithms take a number of steps polynomial in the input size, e.g., the number of digits of the integer to be factored.

Shor's algorithm: how it works

Algorithm 1: ShorAlgorithm(N)

Input: N

Output: p, q such that $N = pq$

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Pick $g \in \mathbb{Z}_N$ at random;

if $\gcd(g, N) \neq 1$ **then**

then **return** $(p = \gcd(g, N), q = N/p)$

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4 Find r such that g^r ≡ 1[N];
5 if r ≡ 0[2] then
6     return gcd(g^{r/2} ± 1, N)
7 else
8     go to 1
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\end{aligned}
$$

Meaning that there is a non-negligible probability that $g^{r/2} \pm 1$ shares non trivial factors with N.

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(source: minutephysics)

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- step 1. $g \leftarrow 127$
- step 2. $r \leftarrow 17388$
- step 3. let us denote $g_p = g^{17388/2} + 1$ and $g_q = g^{17388/2} - 1$
  we have that $\gcd(g_p, N) = 829 =: p$ and $\gcd(g_q, N) = 379 =: q$
  and indeed, $p \cdot q = 829 \times 379 = 314191 = N$

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The complexity to find the *period* of the function $g \mapsto g^x \mod N$ is:

- Classically $\mathcal{O}(N)$
- Quantumly $\mathcal{O}\left(\log(N)^3\right)$. That's an **exponential** speedup!

Quantum period finding

How does it work? Why is it much much faster quantumly?

Fourier Transform is THE tool to analyse frequencies. Fortunately, it has a quantum equivalent: QFT.

Quantum computing allows to provide QFT a superposition of every possible states (assuming enough qubits).

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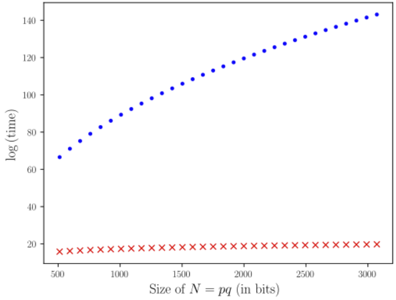

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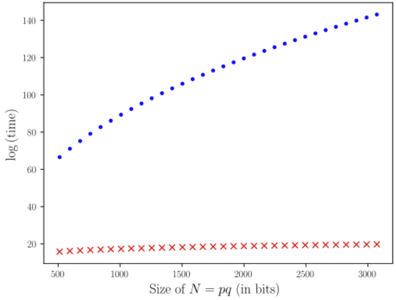

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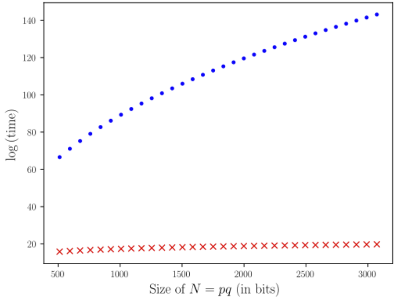

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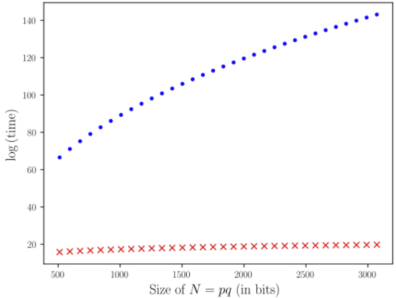

In other words, **security as we know it collapses**...

Grover's algorithm

A fast quantum mechanical algorithm for database search

Lov K. Grover
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Murray Hill NJ 07974
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Summary

Imagine a phone directory containing N names arranged in completely random order. In order to find someone's phone number with a probability of $\frac{1}{2}$, any classical algorithm (whether deterministic or probabilistic) will need to look at a minimum of $\frac{N}{2}$ names. Quantum mechanical systems can be in a superposition of states and simultaneously examine multiple names. By properly adjusting the phases of various operations, successful computations reinforce each other while others interfere randomly. As a result, the desired phone number can be obtained in only $O(\sqrt{N})$ steps. The algorithm is within a small constant factor of the fastest possible quantum mechanical algorithm.

Consequences of Grover's algorithm

(n-entries unsorted) Database search takes $\mathcal{O}(\sqrt{n})$ queries instead of $\mathcal{O}(n)$.

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Consequence over symmetric crypto:

→ The length of the secret key must be **doubled** to preserve the same level of security

Consequence over hash functions:

→ More tricky (depending on the model, the size of the quantum computer, . . .), at least $+33\%$ to preserve the security level

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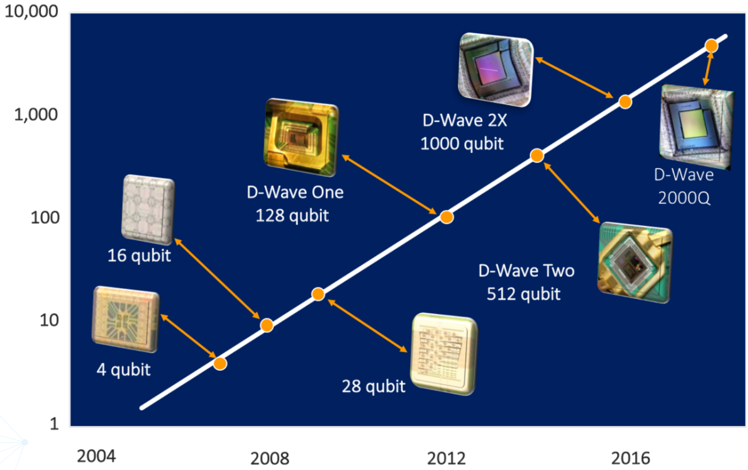
How far are we from a large-scale quantum computer?



A quantum analog to Moore's law: the number of qubits (y-axe) approximately doubles every year (x-axe). (Source: D-Wave)

Large-scale quantum computing: a caveat

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In 2019, the largest quantum computer features 72 qubits (Google).

Hot news!

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TECH • QUANTUM COMPUTING

Google Claims 'Quantum Supremacy,' Marking a Major Milestone in Computing

By Robert Hackett September 20, 2019

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Google's supposed milestone achievement became public last month when a preprint scientific paper accidentally leaked on the website of NASA, a collaborator, as *Fortune* reported at the time. Google has said nothing about the potentially historic experiment since then, lending credence to whispers that its researchers are bound to silence under the terms of a news embargo by a major science journal, unable to disclose more information until a certain date which is presumed to be imminent.

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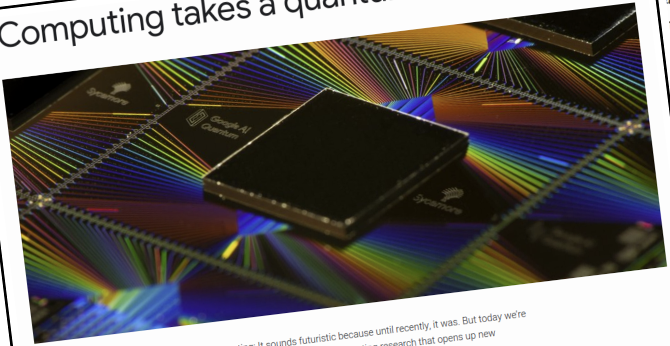
Computing takes a quantum leap forward



Hartmut Neven
Engineering Director, Google AI Quantum Team

Published Oct 23, 2019

Quantum computing: It sounds futuristic because until recently, it was. But today we're marking a major milestone in quantum computing research that opens up new possibilities for this technology.

Unlike classical computing, which runs everything from your cell phone to a supercomputer, quantum computing is based on the properties of quantum mechanics. As a result, quantum computers could potentially solve problems that would be too difficult or even impossible for classical computers—like designing better batteries,

What is quantum supremacy?

What is quantum supremacy?

Quantum supremacy refers to the moment where a functional quantum computer can effectively solve a problem that is not solvable (within decent time frame, *e.g.* 100 years) with any (super) computer.

Wha

Quantum supre ntum computer can
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Why is Google's quantum supremacy experiment impressive?

Asked 13 days ago Active 11 days ago Viewed 12k times

129

21

In the Nature paper published by Google, they say,

> To demonstrate quantum supremacy, we compare our quantum processor against state-of-the-art classical computers in the task of sampling the output of a pseudo-random quantum circuit. Random circuits are a suitable choice for benchmarking because they do not possess structure and therefore allow for limited guarantees of computational hardness. We design the circuits to entangle a set of quantum bits (qubits) by repeated application of single-qubit and two-qubit logical operations. Sampling the quantum circuit's output produces a set of bitstrings, for example {0000101, 1011100, …}. Owing to quantum interference, the probability distribution of the bitstrings resembles a speckled intensity pattern produced by light interference in laser scatter, such that some bitstrings are much more likely to occur than others. Classically computing this probability distribution becomes exponentially more difficult as the number of qubits (width) and number of gate cycles (depth) grow.

So, from what I can tell, they configure their qubits into a pseudo-randomly generated circuit, which, when run, puts the qubits into a state vector that represents a probability distribution over 2^{53} possible states of the qubits, but that distribution is intractable to calculate, or even estimate via sampling using a classical computer simulation. But they sample it by "looking" at the state of the qubits after running the circuit many times.

Isn't this just an example of creating a system whose output is intractable to calculate, and then "calculating" it by simply observing the output of the system?

It sounds similar to saying:

> If I spill this pudding cup on the floor, the exact pattern it will form is very chaotic, and intractable for any supercomputer to calculate. But I just invented a new special type of computer: this pudding cup. And I'm going to do the calculation by spilling it on the floor and observing the result. I have achieved pudding supremacy.

What i… ?

Quantum suprem… …antum computer can effectively solve a… …e frame, *e.g.* 100 years) with any (super) comp…

Why is Google's quantum supremacy experiment impressive?

Asked 13 days ago Active 11 days ago Viewed 12k times

129

21

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What is quantum supremacy?

Quantum supremacy refers to the moment where a functional quantum computer can effectively solve a problem that is not solvable (within decent time frame, *e.g.* 100 years) with any (super) computer.

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It however remains impressive, since no regular computer can do that efficiently. A bit weaker than supremacy is “quantum advantage”, where a quantum computer simply performs better than any computer.

Open challenges towards quantum computing

More work in required to embrace a large scale quantum computer:

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- developing quantum error-correcting codes for error-free quantum computing

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Still, a Sword of Damocles hanging over our heads, and **now** is the time for designing **quantum-safe** alternatives.

Outline

1 What you've learnt so far (should have)

2 Classical vs Quantum computing

3 Two noticeable quantum algorithms (and their impact over cryptography)

4 State-of-the-art quantum computers

5 Quantum safe alternatives

Clarification

Quelles sont les alternatives à la cryptographie classique en présence d'un adversaire possédant un ordinateur quantique puissant ?

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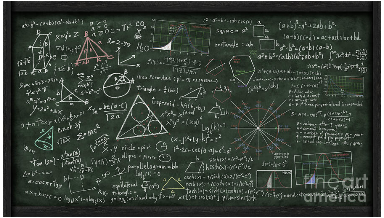
- Cryptographie post-quantique



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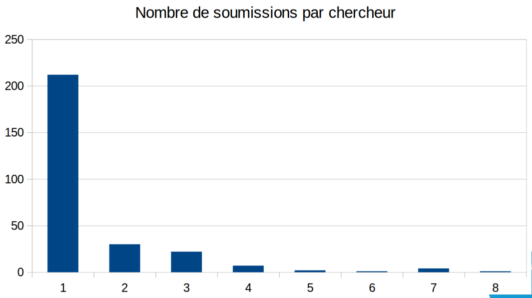

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- Au 13/02 : 26 acceptées au 2nd tour.

Hot topic!

	Signatures	KEM/Encryption	Overall
Lattice-based	4	24	28
Code-based	5	19	24
Multi-variate	7	6	13
Hash-based	4		4
Other	3	10	13
Total	23	59	82

Submissions available at:

- https://csrc.nist.gov/Projects/post-quantum-cryptography/Post-Quantum-Cryptography-Standardization
- https://www.safecrypto.eu/pqclounge/

source: Dustin Moody, NIST

Hot topic!

Below is a timeline of major events with respect to the NIST PQC Standardization Process.

- April 2-3, 2015 — Workshop on Cybersecurity in a Post-Quantum World, NIST, Gaithersburg, MD
- February 24, 2016 — PQC Standardization: Announcement and outline of NIST's Call for Submissions presentation given at PQCrypto 2016
- April 28, 2016 — NISTIR 8105, Report on Post-Quantum Cryptography, released
- August 2, 2016 — Federal Register Notice - Proposed Requirements and Evaluation Criteria announced for public comment
- December 20, 2016 — Federal Register Notice – Announcing Request for Nominations for Public-Key Post-Quantum Cryptographic Algorithms
- November 30, 2017 — Submission Deadline for NIST PQC Standardization Process
- December 20, 2017 — First-Round Candidates were announced. The public comment period on the first-round candidates began.
- April 11-13, 2018 — First NIST PQC Standardization Conference, Ft. Lauderdale, FL
- January 30, 2019 — The First Round ended and the Second Round began. Second-Round candidates announced. The public comment period on the second-round candidates began.
- March 15, 2019 — Deadline for updated submission packages for the Second Round
- August 22-24, 2019 — 2nd NIST PQC Standardization Conference, Santa Barbara, CA

source:
NIST IR 8240

Hot topic!

Timeline

**This is a tentative timeline, provided for information, and subject to change.*

Date	
Feb 24-26, 2016	NIST Presentation at PQCrypto 2016: *Announcement and outline of NIST's Call for Submissions (Fall 2016)*, *Dustin Moody*
April 28, 2016	NIST releases NISTIR 8105, Report on Post-Quantum Cryptography
Dec 20, 2016	Formal Call for Proposals
Nov 30, 2017	Deadline for submissions
Dec 4, 2017	NIST Presentation at AsiaCrypt 2017: *The Ship Has Sailed: The NIST Post-Quantum Crypto "Competition"*, *Dustin Moody*
Dec 21, 2017	Round 1 algorithms announced (69 submissions accepted as "complete and proper")
Apr 11, 2018	NIST Presentation at PQCrypto 2018: *Let's Get Ready to Rumble - The NIST PQC "Competition"*, *Dustin Moody*
April 11-13, 2018	First PQC Standardization Conference - Submitter's Presentations
January 30, 2019	Second Round Candidates announced (26 algorithms)
March 15, 2019	Deadline for updated submission packages for the Second Round
May 8-10, 2019	NIST Presentation at PQCrypto 2019: Round 2 of the NIST PQC "Competition" - What was NIST Thinking? (Spring 2019), *Dustin Moody*
August 22-24, 2019	Second PQC Standardization Conference
2020/2021	Round 3 begins or select algorithms
2022/2024	Draft Standards Available

Outline

5 Quantum safe alternatives
- Lattice-based cryptography
- Hash-based cryptography
- Code-based cryptography

Definitions

Lattice

An m-dimensional lattice is a discrete subgroup of $\mathbb{R}^m$. Formally, if $\mathbf{b}_1, \ldots, \mathbf{b}_n \in \mathbb{R}^m$, the lattice $\Lambda(\mathbf{b}_1, \ldots, \mathbf{b}_n)$ is the set

$$\Lambda = \{\sum_{i=1}^{n} x_i \mathbf{b}_i ; x_i \in \mathbb{Z}\} \subset \mathbb{R}^m$$

Vocabulary

- rank n (main security parameter)
- dimension m ($m = \mathcal{O}(n \cdot \log n)$)
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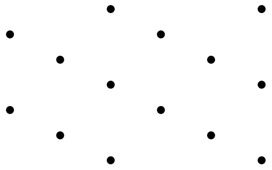

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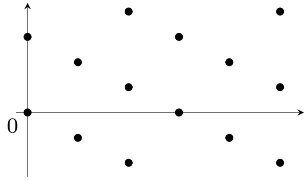

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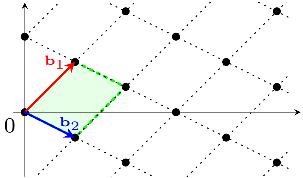

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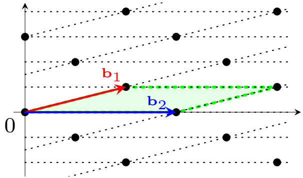

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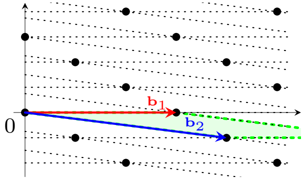

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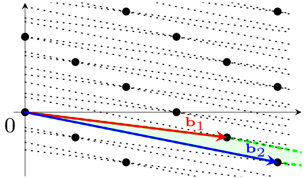

Matrix Representation and q-ary Lattices

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q-ary Lattices

Let $\mathbf{B} = (\mathbf{b}_1 | \cdots | \mathbf{b}_n) \in \mathbb{Z}_q^{m \times n}$ for some prime q, and let

$$\Lambda_q(\mathbf{B}) = \{\mathbf{B} \cdot \mathbf{x} \mod q : \mathbf{x} \in \mathbb{Z}^n\}, \text{ and}$$

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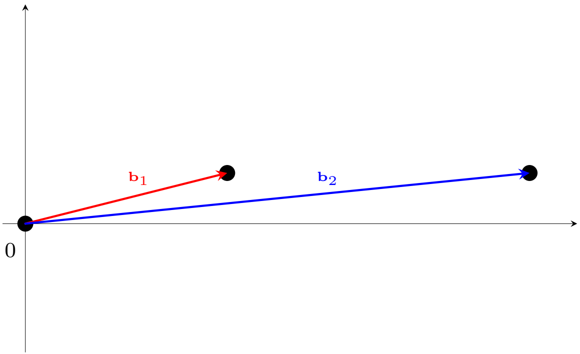

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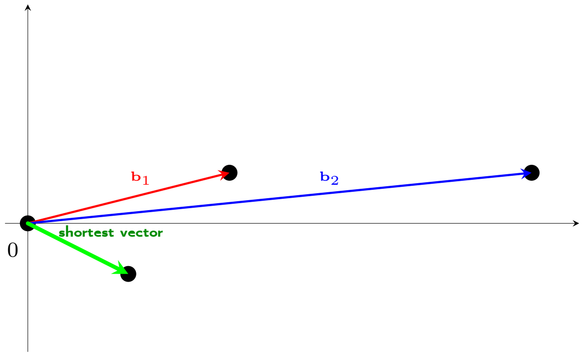

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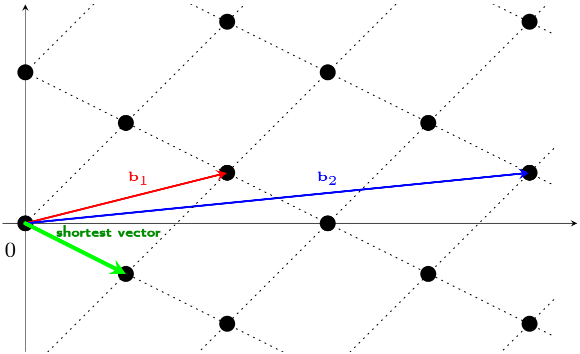

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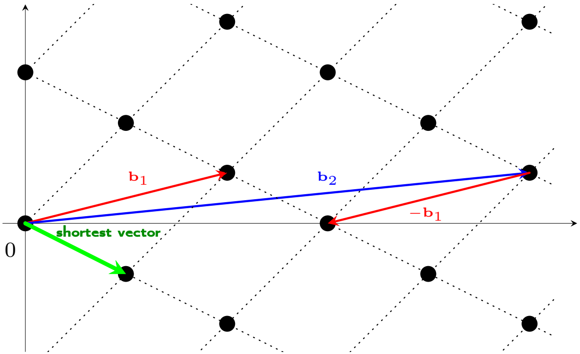

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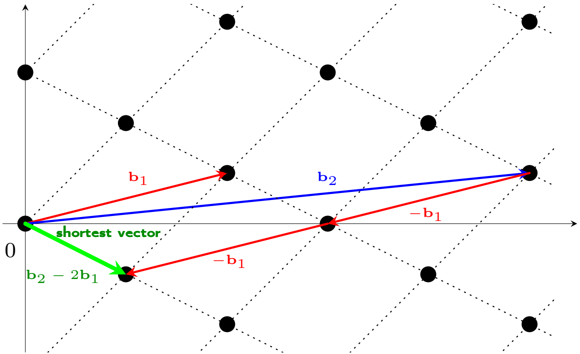

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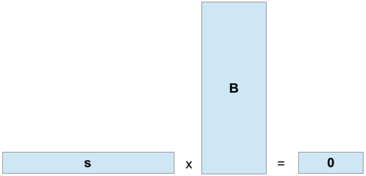

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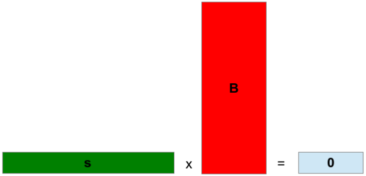

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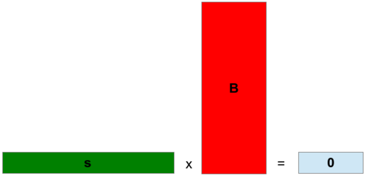
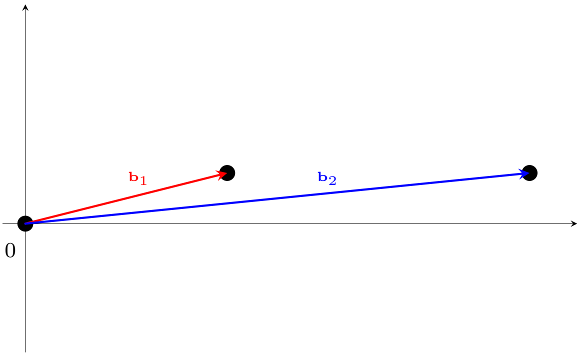


Relationship to Lattices

Solving **SIS** in random lattices $\mathbf{B}$ is "close" to solving **SVP** in $\Lambda_q^{\perp}(\mathbf{B})$

Hard problems: the Closest Vector Problem



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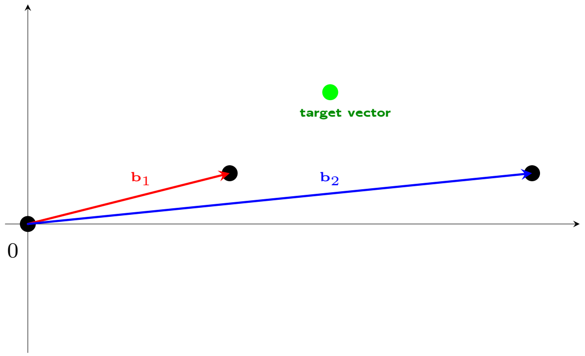

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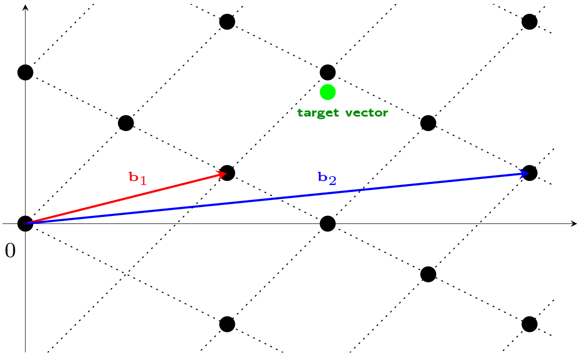

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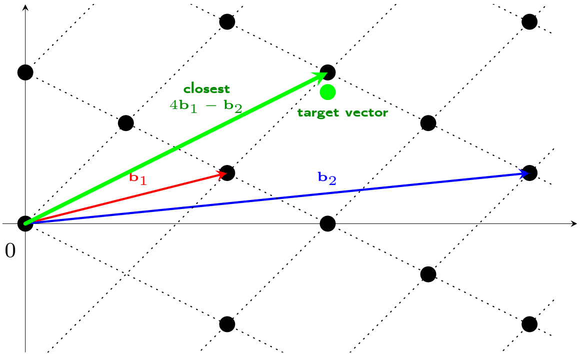


Hard problems: the Learning with Errors

The Learning with Errors (LWE) problem was defined by Regev.
Given $(\mathbf{A}, \mathbf{c})$ with $\mathbf{c} \in \mathbb{Z}_q^m$, $\mathbf{A} \in \mathbb{Z}_q^{mn}$, $\mathbf{s} \in \mathbb{Z}_q^n$ and small $\mathbf{e} \in \mathbb{Z}^m$ is

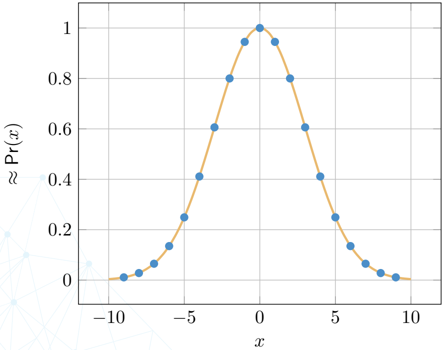
$$\begin{pmatrix} \\ \\ \mathbf{c} \\ \\ \\ \end{pmatrix} = \begin{pmatrix} \leftarrow & n & \rightarrow \\ & & \\ & \mathbf{A} & \\ & & \\ & & \end{pmatrix} \cdot \begin{pmatrix} \mathbf{s} \end{pmatrix} + \begin{pmatrix} \\ \\ \mathbf{e} \\ \\ \\ \end{pmatrix}$$

or $\mathbf{c} \leftarrow_{\$} \mathcal{U}\left(\mathbb{Z}_q^m\right)$.

Relation to other problems

Solving LWE in random lattices is close to to solving CVP in $\Lambda_q(\mathbf{B})$.

Parameters



- Parameters are:
  - dimension n,
  - modulus q (e.g. $q \approx n^2$),
  - noise size α (e.g. $\alpha q \approx \sqrt{n}$),
  - number of samples m.
- Elements of $\mathbf{A}, \mathbf{s}, \mathbf{e}, \mathbf{c}$ are in $\mathbb{Z}_q$.
- $\mathbf{e}$ is sampled from χ_α, a discrete Gaussian with width

$$\sigma = \frac{\alpha q}{\sqrt{2\pi}}.$$

LBC: what about encryption

In 2005, Regev proposed a lattice-based encryption scheme.

KeyGen

Given n, m, q, α, generate $\mathbf{e} \leftarrow D_\alpha$ output $sk = \mathbf{s} \in \{-1, 0, 1\}^n$ and $pk = (\mathbf{A}, \mathbf{b})$ where $\mathbf{b} = \mathbf{A}\mathbf{s} + \mathbf{e}$

Decrypt

Compute $\ell = v - \mathbf{u}^\top \mathbf{s}$. If ℓ is close to 0 output 0, otherwise, output 1.

Encrypt $m \in \{0, 1\}$

$\mathbf{r} \leftarrow \{0, 1\}$ and output $\mathbf{u} = \mathbf{r}^\top \mathbf{A}$ and $v = \mathbf{r}^\top \mathbf{b} + \lfloor q/2 \rfloor \times m$

Regev's cryptosystem relies on a lattice-related problem called LWE.
Notice that there exist other cryptosystems that improve upon this one.

Lattice problems

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- hard given a "bad" basis (constituted of long and almost parallel vectors) $\rightarrow$ pk
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All these problems do not seem hard in dimension 2...

Question: how hard is it to obtain a good basis given a bad basis?

Best known attacks: lattice reduction

Given a bad basis $\mathcal{B}$, find linear combinations of its vector to obtain a reduced and almost orthogonal good basis $\mathcal{B}'$.

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LLL [LLL82] solution:

- Replace $\frac{\langle \mathbf{b}_j^*, \mathbf{b}_i \rangle}{\langle \mathbf{b}_j^*, \mathbf{b}_j^* \rangle}$ by $\left\lfloor \frac{\langle \mathbf{b}_j^*, \mathbf{b}_i \rangle}{\langle \mathbf{b}_j^*, \mathbf{b}_j^* \rangle} \right\rceil$, the nearest integer

Best known attacks: lattice reduction

LLL algorithm:

- Polynomial-time algorithm, but...
- Exponential approximation factor (the resulting basis $\mathcal{B}'$ is not that good)...

Other algorithms that trade memory/time for quality exist:

- blockwise generalization of LLL: BKZ
- Sieving
- Enumeration

They are out of the scope of this course.

Security Level

- $\delta = \left(\frac{\lambda_1}{\det(\Lambda)^{1/n}}\right)^{1/n}$ [CN11]

BKZ 2.0: Better Lattice Security Estimates

Yuanmi Chen and Phong Q. Nguyen

[1] ENS, Dept. Informatique, 45 rue d'Ulm, 75005 Paris, France.
http://www.eleves.ens.fr/home/ychen/
[2] INRIA and ENS, Dept. Informatique, 45 rue d'Ulm, 75005 Paris, France.
http://www.di.ens.fr/~pnguyen/

Abstract. The best lattice reduction algorithm known in practice for high dimension is Schnorr-Euchner's BKZ: all security estimates of lattice cryptosystems are based on NTL's old implementation of BKZ. However, recent progress on lattice enumeration suggests that BKZ and its NTL implementation are no longer optimal, but the precise impact on security estimates was unclear. We assess this impact thanks to extensive experiments with BKZ 2.0, the first state-of-the-art implementation of BKZ incorporating recent improvements, such as Gama-Nguyen-Regev pruning. We propose an efficient simulation algorithm to model the behaviour of BKZ in high dimension with high blocksize ≥ 50, which can predict approximately both the output quality and the running time, thereby revising lattice security estimates. For instance, our simulation suggests that the smallest NTRUSign parameter set, which was claimed to provide at least 93-bit security against key-recovery lattice attacks, actually offers at most 65-bit security.

Security Level

- $\delta = \left(\frac{\lambda_1}{\det(\Lambda)^{1/n}}\right)^{1/n}$ [CN11]
- "Exact" bitlevel correpsondance [LP11]

k	δ
80	1.00783
100	1.00696
128	1.00602

$$\log_2(\delta) := \frac{1.8}{\log_2\left(\frac{T_{BKZ}(\delta)}{2^{30}}\right) + 110} = \frac{1.8}{k - 30 + 110} = \frac{1.8}{k + 80}$$

Better Key Sizes (and Attacks) for
LWE-Based Encryption

Richard Lindner* Chris Peikert†

November 30, 2010

Abstract

We analyze the concrete security and key sizes of theoretically sound lattice-based encryption schemes based on the "learning with errors" (LWE) problem. Our main contributions are: (1) a new lattice attack on LWE that combines basis reduction with an enumeration algorithm admitting a time/success tradeoff, which performs better than the simple distinguishing attack considered in prior analyses; (2) concrete parameters and security estimates for an LWE-based cryptosystem that is more compact and efficient than the well-known schemes from the literature. Our new key sizes are up to 10 times smaller than prior examples, while providing even stronger concrete security levels.

Security Level

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- "Exact" bitlevel correpsondance [LP11]
- Depends on the algorithm

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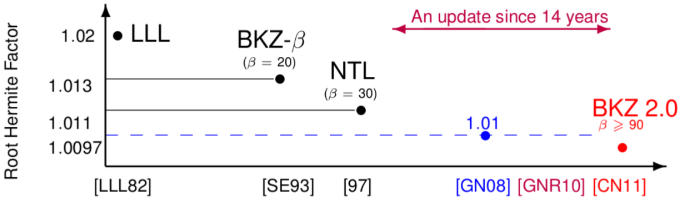

NTRUSign: lattice-based signature

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History

- Originally NSS [HPS01]
- NTRUSign [HPSW02]

NSS: An NTRU Lattice-Based Signature Scheme

Jeffrey Hoffstein, Jill Pipher, and Joseph H. Silverman

NTRU Cryptosystems, Inc., 5 Burlington Woods,
Burlington, MA 01803 USA,
jhoff@ntru.com, jpipher@ntru.com, jhs@ntru.com

Abstract. A new authentication and digital signature scheme called the NTRU Signature Scheme (NSS) is introduced. NSS provides an authentication/signature method complementary to the NTRU public key cryptosystem. The hard lattice problem underlying NSS is similar to the hard problem underlying NTRU, and NSS similarly features high speed, low footprint, and easy key creation.

NTRUSign: lattice-based signature

History

- Originally NSS [HPS01]
  Quickly broken [GS02]
- NTRUSign [HPSW02]

Cryptanalysis of the Revised NTRU Signature Scheme

Craig Gentry[1] and Mike Szydlo[2]

[1] DoCoMo USA Labs, San Jose, CA, USA,
cgentry@docomolabs-usa.com
[2] RSA Laboratories, Bedford, MA, USA,
mszydlo@rsasecurity.com

Abstract. In this paper, we describe a three-stage attack against Revised NSS, an NTRU-based signature scheme proposed at the Eurocrypt 2001 conference as an enhancement of the (broken) proceedings version of the scheme. The first stage, which typically uses a transcript of only 4 signatures, effectively cuts the key length in half while completely avoiding the intended hard lattice problem. After an empirically fast second stage, the third stage of the attack combines lattice-based and congruence-based methods in a novel way to recover the private key in polynomial time. This cryptanalysis shows that a passive adversary observing only a few valid signatures can recover the signer's entire private key. We also briefly address the security of NTRUSign, another NTRU-based signature scheme that was recently proposed at the rump session of Asiacrypt 2001. As we explain, some of our attacks on Revised NSS may be extended to NTRUSign, but a *much* longer transcript is necessary. We also indicate how the security of NTRUSign is based on the hardness of several problems, not solely on the hardness of the usual NTRU lattice problem.

NTRUSign: lattice-based signature

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$$\mathbf{f},\ \mathbf{g} = \begin{cases} d \text{ coefficients } & +1 \\ N-d \text{ coefficients } & 0 \end{cases}$$

$$\mathbf{F},\ \mathbf{G} \text{ st. } \mathbf{f} * \mathbf{G} - \mathbf{F} * \mathbf{g} = q$$

$$\mathbf{h} = \mathbf{g} * \mathbf{f}^{-1} \overset{\$}{\leftsquigarrow} \mathcal{R}_q = \mathbb{Z}_q[X]/\langle X^N + 1\rangle$$

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$$\mathbf{P} = \left(\begin{array}{c|c} \mathbf{1} \circlearrowleft & \mathbf{h} \circlearrowleft \\ \hline \mathbf{0} \circlearrowleft & \mathbf{q} \circlearrowleft \end{array} \right) \qquad \mathbf{S} = \left(\begin{array}{c|c} \mathbf{f} \circlearrowleft & \mathbf{g} \circlearrowleft \\ \hline \mathbf{F} \circlearrowleft & \mathbf{G} \circlearrowleft \end{array} \right)$$

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NTRU lattice: $\Lambda_{\mathbf{h},q} = \{(\mathbf{u}, \mathbf{u} * \mathbf{h} \mod q), \mathbf{u} \in \mathcal{R}_q\}$

NTRUSign

Sign

Given $\mu \in \{0,1\}^*$ to sign:

- Define $\mathbf{m} = \mathcal{H}(\mu)$
- Solve CVP with target $(\mathbf{0}, \mathbf{m})$ and good basis $\mathbf{S}$

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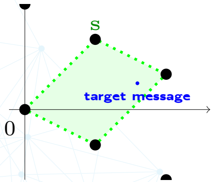

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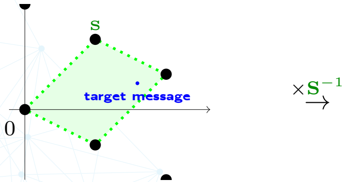

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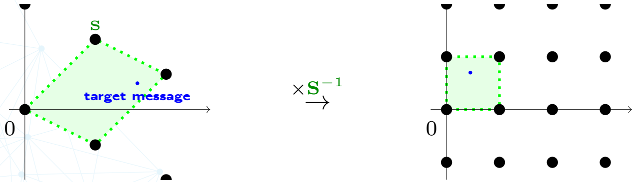

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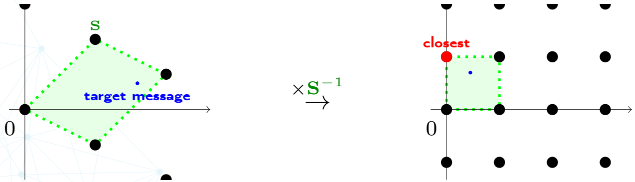

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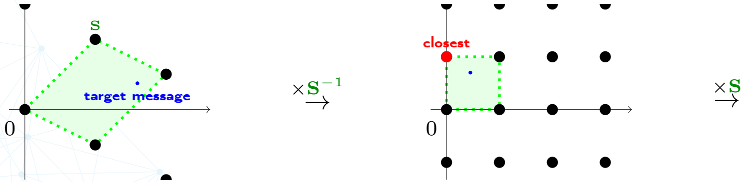

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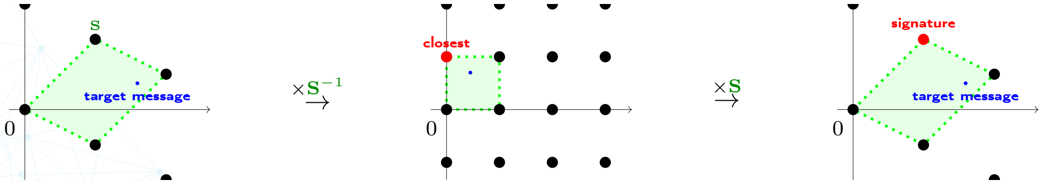

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Given the signature s, check:

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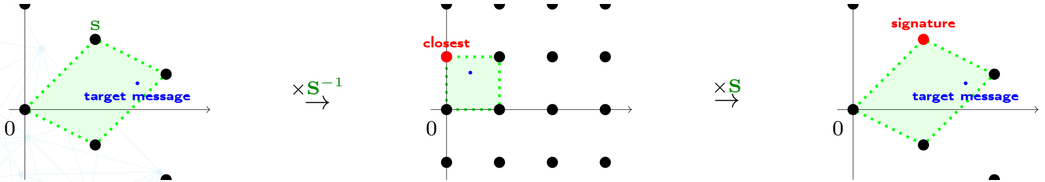

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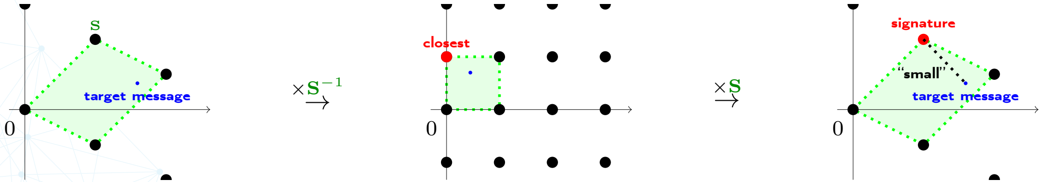

NTRUSign

Signature Size (in bits)

security	80	112	128	160
NTRUSign	1256	1576	1784	2367
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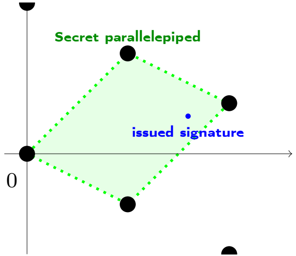
But...

Problem : Not Zero-Knowledge

Key-recovery attacks

- Only a few signatures for original scheme [NR06]
- And a little more to break coutermeasures [DN12]



Number of signature issued : 1

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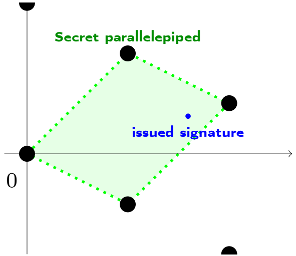

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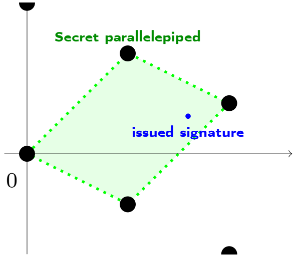

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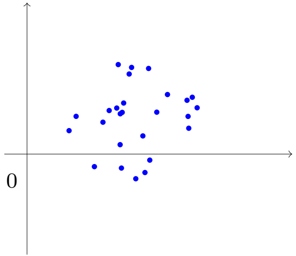

Number of signatures issued : 25

Problem : Not Zero-Knowledge

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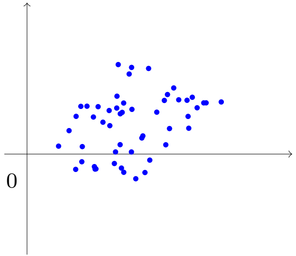

Number of signatures issued : 50

Problem : Not Zero-Knowledge

Key-recovery attacks

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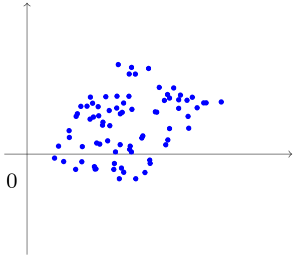

Number of signatures issued : 75

Problem : Not Zero-Knowledge

Key-recovery attacks

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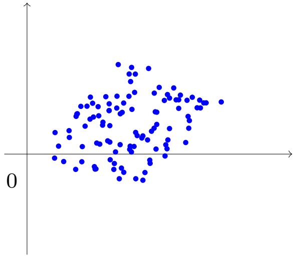

Number of signatures issued : 100

Problem : Not Zero-Knowledge

Key-recovery attacks

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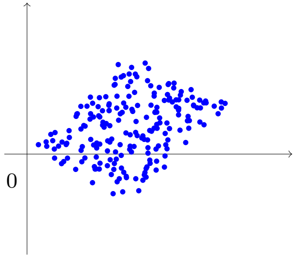

Number of signatures issued : 200

Problem : Not Zero-Knowledge

Key-recovery attacks

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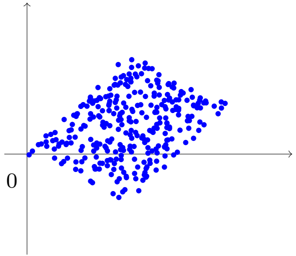

Number of signatures issued : 300

Problem : Not Zero-Knowledge

Key-recovery attacks

- Only a few signatures for original scheme [NR06]
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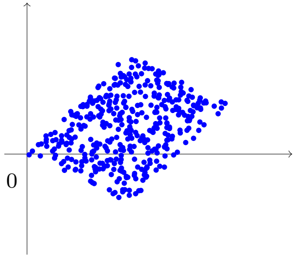

Number of signatures issued : 400

Problem : Not Zero-Knowledge

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- Only a few signatures for original scheme [NR06]
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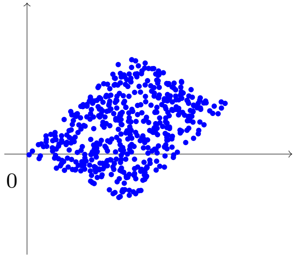

Number of signatures issued : 500

Problem : Not Zero-Knowledge

Key-recovery attacks

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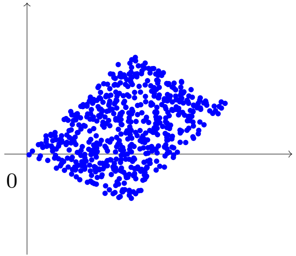

Number of signatures issued : 600

Problem : Not Zero-Knowledge

Key-recovery attacks

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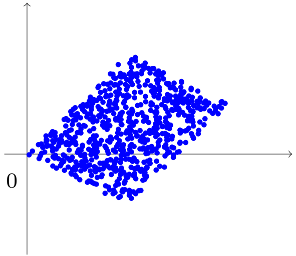

Number of signatures issued : 700

Problem : Not Zero-Knowledge

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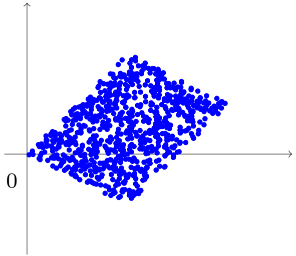

Number of signatures issued : 800

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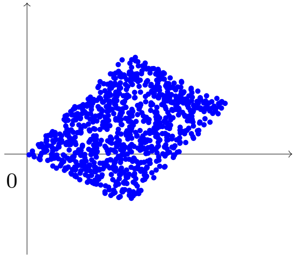

Number of signatures issued : 900

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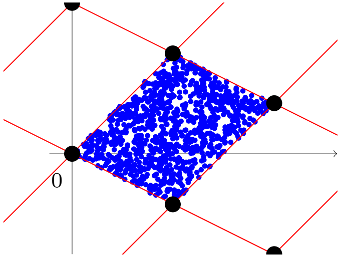

Number of signatures issued : 1000

Secure lattice based signatures [Lyu12]

KeyGen

- Secret key : $\mathbf{S} \xleftarrow{\$} \{-d, \ldots, 0, \ldots, d\}^{m \times k}$
- Public key : $\mathbf{A} \xleftarrow{\$} \mathbb{Z}_q^{n \times m}$ and $\mathbf{T} = \mathbf{A} \cdot \mathbf{S} \in \mathbb{Z}_q^{n \times k}$

Sign

First stage [Finding pre-image]

- map μ to a space element $\mathbf{c}$
- $\mathbf{Sc}$ is a short pre-image of $\mathbf{Tc}$

Second stage [Hiding pre-image]

- Add gaussian noise $\mathbf{y}$ to $\mathbf{Sc}$
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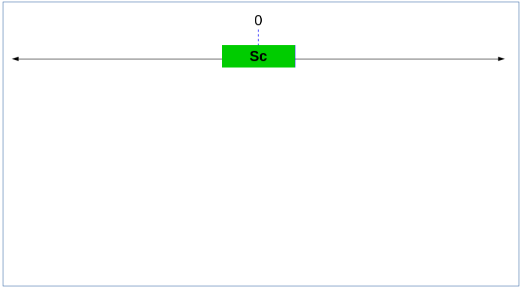

Verify

Given $(\mathbf{z}, \mathbf{c})$, check that :

- $H(\underbrace{\mathbf{Az} - \mathbf{Tc}}_{\mathbf{A}(\mathbf{Sc}+\mathbf{y})-\mathbf{ASc}}, \mu) = \mathbf{c}$ $\rightarrow$ it is a lattice vector
- $\|\mathbf{z}\| \leq \eta\sigma\sqrt{m}$ $\rightarrow$ it has reasonnable norm

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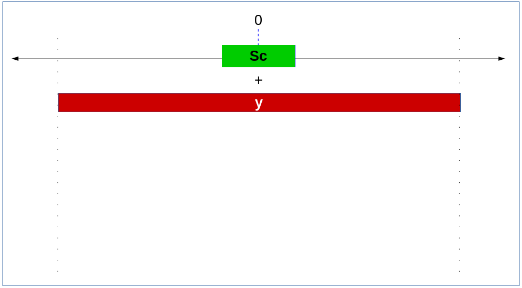

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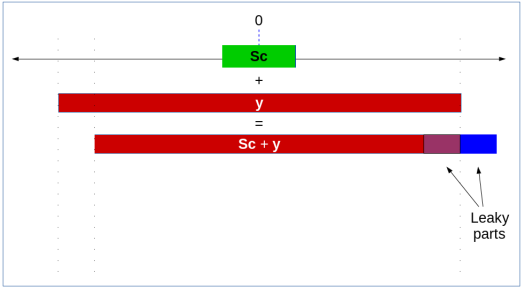

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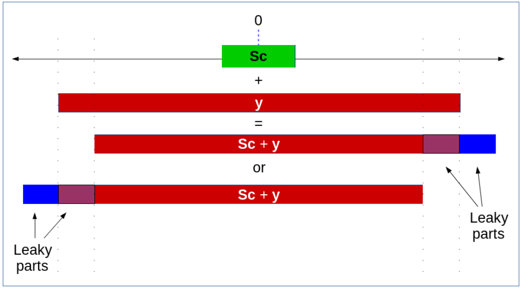

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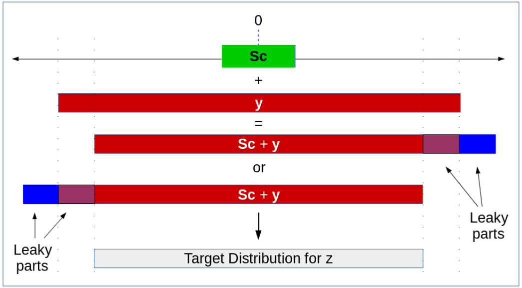

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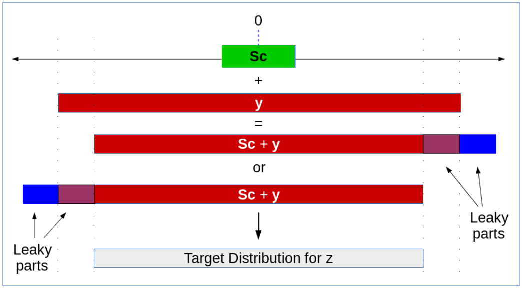

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Sets of parameters

100 bits of security

n	512	512	512	512	512
m	8,786	8,139	3,253	1,024	1,024
k	80	512	512	512	512
$\log_2(q)$	27	25	33	18	26
d	1	1	31	1	31
M (retries)	2.72	2.72	2.72	7.4	7.4
≈ sign size	163,000	142,300	73,000	14,500	19,500
≈ pk size	2^{20}	$2^{22.5}$	2^{23}	$2^{19.5}$	$2^{21.5}$
≈ sk size	2^{20}	$2^{22.5}$	2^{23}	$2^{22.1}$	$2^{22.7}$

More recent proposals achieve better security, parameters and performances (along with other nice features).

Outline

5 Quantum safe alternatives
- Lattice-based cryptography
- Hash-based cryptography
- Code-based cryptography

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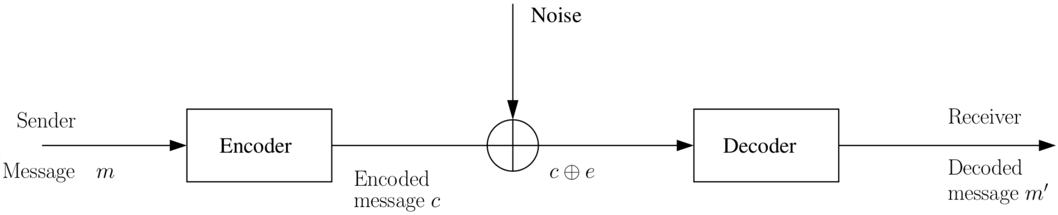
Coding theory

Coding theory is the science of (efficiently) adding redundancy to information in order to detect/correct errors that could occur during transmission.



Codes Correcteurs

Théorie des Codes

- Ajout de redondance à l'information
- En cas d'erreur(s), permet soit :
  - De détecter l'erreur $\Rightarrow$ Renvoi
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Exemple basique : code à 3-répétition

- Alice souhaite envoyer $1 \cdot 0 \cdot 1$
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Métrique de Hamming

$\mathbf{u}, \mathbf{v} \in \mathbb{F}_q^n$, disons $\mathbb{F}_5^7$

$\mathbf{u} =$	3	3	2	4	4	5	2
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- Attaques plus directes qu'en métrique rang

Code-based cryptography (CBC)

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Des façons de rajouter de la redondance à l'information utile, afin d'être capable de détecter — voire corriger — d'éventuelles erreurs lors de la transmission.

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Exemple : le code à répétition

Message à envoyer		1			0			1	
Encodage	1	1	1	0	0	0	1	1	1
Message reçu	0	1	1	0	1	0	1	1	0
Message décodé		1			0			1	

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Ce code est particulièrement mauvais (bien qu'utile pédagogiquement parlant) :

- dimension : $k = 1$
- longueur : $n = 3$
- distance minimale : $d = 3$
- capacité de détection : $d - 1 = 2$ erreurs
- capacité de correction : $\left\lfloor \frac{d-1}{2} \right\rfloor = 1$ erreur
- rendement $\frac{k}{n} = \frac{1}{3}$.

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Un code $\mathcal{C}$ est entièrement définit par sa matrice génératrice $\mathbf{G}$:

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$$\text{exemple : } wt\left((0,1,0,0,1,0,1,0)\right) = ?$$

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Problème modifié

Soit $\mathbf{s} \in \mathbb{F}_2^{n-k}$ et $\mathbf{H} \in \mathbb{F}_2^{(n-k)\times n}$. Trouver $\mathbf{x} \in \mathbb{F}_2^n$ tel que $\mathbf{H}\mathbf{x}^\top = \mathbf{s}$, **et $\mathbf{x}$ de poids relativement faible**.

Le problème devient *NP*-difficile [?].

(Traduction: il devient cryptographiquement intéressant)

Code-based cryptography (CBC)

Cryptosystème de McEliece [?]

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matrice inversible $\mathbf{S} \in \mathbb{F}_2^{k \times k}$

matrice permutation $\mathbf{P} \in \mathbb{F}_2^{n \times n}$

message $\mathbf{m} \in \mathbb{F}_2^{k}$

$$\xrightarrow{\tilde{\mathbf{G}}=\mathbf{SGP}, n, k, t}$$

$\mathbf{e} \in \mathbb{F}_2^{n}$ tel que $wt(\mathbf{e}) \leq t$

$$\xleftarrow{\mathbf{c}}$$

$$\mathbf{c} = \mathbf{m}\tilde{\mathbf{G}} + \mathbf{e}$$

$$\tilde{\mathbf{c}} = \mathcal{D}_{\mathbf{G}}\left(\mathbf{cP}^{-1}\right) = \mathcal{D}_{\mathbf{G}}\left(\mathbf{mSG} + \mathbf{eP}^{-1}\right)$$
$$\mathbf{m} = \tilde{\mathbf{c}}\mathbf{S}^{-1}$$

CBC : un exemple

Soit $\mathcal{C}$ le code (de Hamming) admettant pour matrice de parité $\mathbf{H}$:

$$\mathbf{H} = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$$

Soit $\mathbf{s} = (1, 0, 0, 0, 1, 1, 1)$ le mot reçu. Quel était le message envoyé ?

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Quasi-Cyclic Moderate Density Parity-Check Codes

KeyGen

Sample $\mathbf{h}_0, \mathbf{h}_1 \leftarrow \mathbb{F}_2^r$ of small weight w, $\mathbf{h}_0$ inversible. Compute $\mathbf{h} = \mathbf{h}_1 \mathbf{h}_0^{-1}$.

$$\mathbf{H}_{\text{secret}} = \left(\begin{array}{c|c} \mathbf{h}_0 & \mathbf{h}_1 \\ \circlearrowright & \circlearrowright \end{array} \right)$$

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As for McEliece, $\mathbf{e}$ of weight t,

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Chiffrement OK. Existe-t-il un algo de signature aussi simple ?

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$$\mathbf{z} = \left(\begin{array}{cccc|cccc} 1 & 0 & \dots & 0 & c_0 & c_1 & \dots & c_{n-1} \\ 0 & 1 & \dots & 0 & c_{n-1} & c_0 & \dots & c_{n-2} \\ \vdots & & \ddots & \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & 1 & c_1 & c_2 & \dots & c_0 \end{array}\right) \cdot \begin{pmatrix} \mathbf{y} \\ \mathbf{x} \end{pmatrix}$$

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Claimed security	Persichetti's OTS parameters				xBF parameters		Verification	Cryptanalysis
	n	w_1	w_2	δ	τ	N	t_{verify} (ms)	t_{break} (ms)
80	4801	90	100	10	7	5	22.569	165.459
	3072	85	85	7	5	5	14.271	68.858
128	9857	150	200	12	9	10	99.492	453.680
	6272	125	125	10	7	10	42.957	288.442

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D'autres schémas de signature (plus complexes à exposer) existent, et ne souffrent pas de ce type de problème:

- WAVE [?]: `https://eprint.iacr.org/2018/996`
- DURANDAL [?]: `https://eprint.iacr.org/2018/1192` **(métrique rang)**

Rank-based cryptography

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$$\mathbf{v} = \begin{pmatrix} v_0 & v_1 & \ldots & v_{n-1} \end{pmatrix} \in \mathbb{F}_{q^m}^n$$

$$\mathbf{V} = \begin{pmatrix} v_{0,0} & v_{1,0} & \ldots & v_{n-1,0} \\ v_{0,0} & v_{1,0} & \ldots & v_{n-1,0} \\ \vdots & & \ddots & \vdots \\ v_{0,m-1} & v_{1,m-1} & \ldots & v_{n-1,m-1} \end{pmatrix} \in \mathbb{F}_q^{m \times n} \begin{pmatrix} \mathbf{b}_0 \\ \mathbf{b}_1 \\ \vdots \\ \mathbf{b}_{m-1} \end{pmatrix}$$

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Le poids rang du vecteur $\mathbf{v}$ est défini comme le rang de la matrice $\mathbf{V}$

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Exemple avec $q = 5$, $m = 3$, et $n = 3$:

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Distance rang entre deux vecteurs $\mathbf{u}, \mathbf{v} \in \mathbb{F}_{q^m}^n$:

→ $d_R(\mathbf{u}, \mathbf{v}) = \text{rang}(\mathbf{U} - \mathbf{V})$

(symétrie, séparation, inégalité triangulaire)

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En résumé: les attaques en **métrique Rang** ont une complexité **quadratiquement** exponentielle $2^{\mathcal{O}(n^2)}$, contre **simplement** exponentielle $2^{\mathcal{O}(n)}$ pour la **métrique de Hamming**

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